Resource Allocation With Partially Overlapping Filtered Multitone in Cognitive Heterogeneous Networks

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***Abstract*—Partially overlapping tones have recently been offered as a new solution to reduce the other user interference in cognitive heterogeneous networks. By introducing intentional carrier frequency shift, the users can exploit the gaps between the subcarriers. In this letter, we perform game theoretical resource allocation with partially overlapping filtered multitone signaling. To achieve the resource allocation, a secondary base station slides a group of consecutive subcarriers through all available ones and computes the utility for each selected subcarriers. It picks the consecutive ones, which give the highest capacity result. By performing this scheme, the other user interference is reduced significantly with a slightly slower convergence rate.**

***Index Terms*—Partially overlapping filtered multitone, game theory, resource allocation, heterogeneous networks.**

I. INTRODUCTION

PARTIALLY overlapping tones (POTs) have recently gained interests due to the capability of solving the interference problem in cognitive heterogeneous networks (HetNets) by introducing the intentional carrier frequency shift (CFS). By virtue of high spectral efficiency, orthogonal frequency-division multiplexing (OFDM) is extensively utilized in today's systems. However, OFDM is vulnerable against the other user interference due to the selfish behavior of the secondary users (SUs) in cognitive HetNets. At this point, POTs has been proposed as a promising solution for cognitive HetNets against this interference problem [1]. In POTs concept, the gaps between subcarriers are utilized by the SUs. For the case of two SUs, if one SU allocates all available subcarriers, the other SU also uses the same subcarriers by intentionally shifting the carrier frequency as seen in Fig. 1. By performing partial overlapping among SUs, other user interference can significantly be reduced [2].

In the literature, the other user interference has been proposed to be mitigated via utilizing the partially overlapping channels and various game theoretical resource allocation techniques. In [3] and [4], authors exploit the gaps between channels in a partial overlapping manner in WiFi networks to increase the throughput, and hence, reduce the interference. In terms of resource allocation, while in [5], Stackelberg games are utilized in cognitive HetNets, authors in [6] use the supermodular games to perform resource allocation. In [7], potential games are utilized to allocate subcarriers in uncoordinated networks. However, no study considers the resource allocation with POTs.

While one advantageous of POTs is to decrease the other user interference, another advantageous over OFDM can be given as allowing asynchronous transmission, which is also a significant issue in cognitive HetNets, by employing the filtered multitone (FMT) [8]. It is noted that the usage of FMT provides flexibility on adjusting the gap (stemming from the guard-band) between subcarriers by alternating the filter parameters such as roll-off factor. Since this gap is negligible in OFDM scheme, this makes FMT more suitable for POTs concept. In this study, we develop a resource allocation technique for partially overlapping filtered multitone (POFMT) within a game theoretical framework in cognitive HetNets. In downlink, secondary base stations (SBSs) as players perform the resource allocation by searching for the best subcarriers which are constrained to be in consecutive order. To perform subcarrier selection, SBS picks a certain number of subcarriers based on the total need of SU by starting from the first available subcarrier and computes the utility of the selected subcarriers. Then, it shifts the subcarriers intentionally to introduce the CFS and calculates the utility in this position, too. The SBS performs this operation throughout the all available subcarriers. After obtaining the capacity results for every position and subcarriers, SBS selects the one which provides the highest utility. With this scheme, the existence of Nash equilibrium (NE) is proved theoretically and by simulations. As indicated in simulation results, the proposed scheme outperforms OFDM with a slightly slower convergence rate.

The remainder of this letter is organized as follows. In Section II, the system model is introduced with the transmission and reception models. The problem formulation and potential game formulation with the proof of NE convergence are explained in Section III in which the subcarrier and frequency shift ratio (FSR) selection scheme is also introduced. Section IV entails numerical results of the proposed approach. Conclusion is drawn in Section V.

II. SYSTEM MODEL

We consider a downlink scenario with multiple SBSs, where each SBS serves multiple SU in a given area. Each SU allocates the certain number of subcarriers under the assumption that the total number of available resources is higher than the total number of resources that SUs served by a single SBS need. It is assumed that the intentional CFS is performed by the SBS with the orthogonal waveforms.

The transmitted signal of the SBS i, $i \in \mathcal{I}$ which is the total number of SBSs, is defined as

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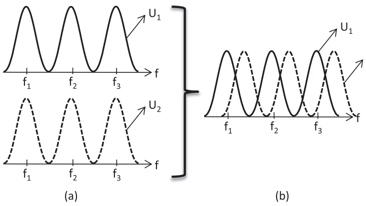

Fig. 1. (a) Both users allocate the same resources at the same time. This leads to fully overlapping case where the highest interference is achieved. (b) U_2 is shifting the carrier frequency. Therefore, users are partially overlapped to each other. This reduces the other user interference.

$$x_i(t;\varphi_i,a_i)=\sum_{m=-\infty}^{\infty}\sum_{k=0}^{N-1}X_{kmi}\,g(t-mT)e^{j2\pi k(f_0+\varphi_i)t}a_{ki}, \tag{1}$$

where N is the total number of subcarriers, f_0 is the subcarrier spacing, *g(t)* is the prototype filter, T is the symbol duration, X_{kmi} is the modulated symbols on the kth subcarrier of mth symbol, a_{ki} is the indicator function, where if the kth subcarrier is used by the SBS i, then $a_{ki}=1$, if not, then $a_{ki}=0$, and $\varphi \in [0,\, f_0]$ is the frequency shift introduced by the SBS i. The aim in CFS is to decrease the other SBS interference.

With a Rayleigh fading channel consideration, the received signal of the SU of the SBS i is given by

$$\begin{aligned}y_i(t;\varphi_i,a_i)&=\int\!\!\int H_i(\tau;\varphi_i)x_i(t-\tau;\varphi_i,a_i)e^{j2\pi(f_0+\varphi_i)t}\mathrm{d}\varphi\mathrm{d}\tau\\&+\sum_{j\in\Im,j\neq i}\int\!\!\int H_j(\tau;\varphi_j)x_j(t-\tau;\varphi_j,a_j)e^{j2\pi(f_0+\varphi_i)t}\mathrm{d}\varphi\mathrm{d}\tau,\end{aligned} \tag{2}$$

where j is the interfering SBSs index, $H_i(\tau;\varphi_i)$ and $H_j(\tau;\varphi_j)$ are the Fourier transformations of time domain channel coefficients of $h_i(t,\tau)$ and $h_j(t,\tau)$, respectively and $w(t)$ is the additive white Gaussian noise. The received symbol can be obtain by projecting the corresponding received filter onto the received signal as

$$\begin{aligned}\tilde{X}_{lni}(\varphi_i,a_i)&=\left\langle y_i(t;\varphi_i,a_i),\,g(t-nT)e^{\mathrm{j}2\pi l(f_0+\varphi_i)t}\right\rangle\\&=\int_t r_i(t,\tau;\varphi_i,a_i)g(t-nT)e^{-\mathrm{j}2\pi l(f_0+\varphi_i)t}\mathrm{d}t\\&+\sum_{\substack{j\in\Im,\\ j\neq i}}\sum_{n=-\infty}^{\infty}\sum_{l=0}^{N-1}\int_t r_j(t,\tau;\varphi_i,a_j)g(t-nT)\\&\times e^{-\mathrm{j}2\pi l(f_0+\varphi_i)t}\mathrm{d}t+w(t),\end{aligned} \tag{3}$$

where $r_i(t,\tau;\varphi_i,a_i)=\int_\tau\int_\varphi H_i(\tau;\varphi_i)x_i(t-\tau;\varphi_i,a_i)\mathrm{d}\varphi\mathrm{d}\tau$.

It is important to mention that for SU to obtain the symbols properly, it needs to employ the same amount of FSR with the SBS. Otherwise, the signal will be taken partially. On the other hand, it becomes advantages when the interference coming from other SBSs is captured. Finally, the signal-to-interference-plus-noise ratio (SINR) of the SU can be expressed as

$$SINR_i=\frac{P_i(\varphi_i,a_i)}{P_j(\varphi_i,a_j)+w_0} \tag{4}$$

where

$$\begin{aligned}P_i(\varphi_i,a_i)=\mathbb{E}\Bigg[\Bigg|\int_t r_i(t,\tau;\varphi_i,a_i)g(t-nT)\\ \times e^{-\mathrm{j}2\pi l(f_0+\varphi_i)t}\mathrm{d}t\Bigg|^2\Bigg],\end{aligned}$$

and

$$\begin{aligned}P_j(\varphi_i,a_j)=\sum_{\substack{j\in\Im,\\ j\neq i}}\sum_{n=-\infty}^{\infty}\sum_{l=0}^{N-1}\mathbb{E}\Bigg[\Bigg|\int_t r_j(t,\tau;\varphi_i,a_j)\\ \times g(t-nT)e^{-\mathrm{j}2\pi l(f_0+\varphi_i)t}\mathrm{d}t\Bigg|^2\Bigg].\end{aligned}$$

III. Problem Formulation

In this study, we consider joint subcarrier and FSR selection to perform the resource allocation with the POFMT. This is a multidimensional optimization problem. However, since finding the optimum solution increases the overhead and the complexity of the system [9], we formulate our problem within the game theoretical structure. Game theory (GT) provides a solution among selfishly behaved players (users) who are interacted with each other. In GT, each player changes its strategy to increase its utility towards its benefits. In this letter, we define the players as the SBSs, strategies as the selection of subcarriers and FSR which are denoted with $s_i=\{\varphi_i,a_i\}$, $s_i\in S_i$ where S_i is the strategy set of player i. Finally, the utility function is defined as the capacity of the ith player and given as

$$U_i(\varphi_i,a_i)=\log_2(1+SINR_i). \tag{5}$$

When the players play with their best responses, they aim at reaching the NE in GT. So, the NE can be defined as

$$\begin{aligned}U_i(s_i^*,s_{-i}^*)\geq U_i(s_i,s_{-i}^*)\\ \forall i\in\Im,\forall s_i,s_{-i}\in S\end{aligned} \tag{6}$$

where s_{-i} indicates the strategies for all players except i, S is the strategy profile of all players and '*' represents the equilibrium point.

A. Subcarrier and FSR Selection Scheme

The selection of subcarrier and FSR is performed with Play&Wait (P'nW) algorithm given in [2]. In this algorithm, players play and wait for random time. For instance, if one second is considered as one iteration, for the case where there are two players, player-1 plays for 3 s and waits for 5 s while player-2 plays for 4 s and waits for 2 s.

In orthogonal frequency-division multiple access (OFDMA) structure, the players can pick any subcarriers among the available ones, i.e., the selected subcarriers don't have to be consecutive. However, when the players perform the subcarrier selection with POFMT, they need to select the ones which are in consecutive order. Otherwise, an intra-cell interference problem may occur. On the other hand, it is a

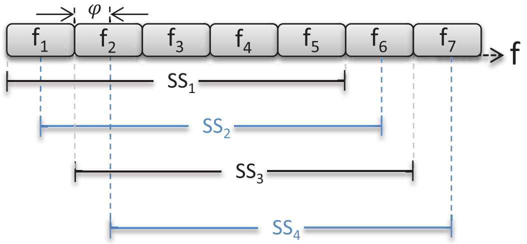

Fig. 2. A player searches for the consecutive subcarriers which give the highest utility by sliding the subcarrier set (SS) through all the available ones.

challenging task to manage the frequency shift in randomly allocated subcarriers among multiple users. Another problem with this randomized scheme is to increase the loss in the spectral efficiency. Because of these drawbacks, we introduce the constraint on the selection of subcarriers in terms of performing it in a consecutive order.

With this constraint, a player searches for the consecutive subcarriers which give the highest utility by sliding the subcarriers through all the available ones as seen in Fig. 2 which is shown for the selection of five subcarriers as an example. The player first picks the subcarriers of f_1 to f_5 and computes the utility for these subcarriers. Then, it introduces the CFS and again, computes the utility for the shifted consecutive subcarriers, i.e., for $(f_1 + \varphi)$ to $(f_5 + \varphi)$. After calculating the utility for all subcarriers with their corresponding CFS, the player selects the ones which provide the highest capacity result. If the same SBS serves to more than one SU, SBS assigns the first SU to its best responses and then, follow the same step for other SUs in a round-robin manner. The detailed explanation of the proposed scheme can be seen in Table I.

B. *Potential Game Formulation*

Potential games are defined with a potential function which shows the unilateral deviation of a player with respect to other players. As given in [10], there are various potential games. Among those, the ordinal potential games are defined as follows.

Definition 1: A game $\mathcal{G}$ is said to be an ordinal potential game if it admits an ordinal potential. A function V is an ordinal potential for $\mathcal{G}$ if, for all, $i \in \mathfrak{I}$ and $\forall s_i, s_i' \in S_i$

$$U_i(s_i', s_{-i}) - U_i(s_i, s_{-i}) > 0 \text{ iff } V(s_i', s_{-i}) - V(s_i, s_{-i}) > 0 \tag{7}$$

where s_i' indicate the deviation of the strategy of the ith player.

NE existence is guaranteed with ordinal potential games [10]. In this study, we define the potential function similar to [11] as

$$V(S) = \log_2\left(\sum_{b\in\mathfrak{I}} P_b(\varphi_b, a_b) + w_0\right) \tag{8}$$

where $S = (s_i, s_{-i})$. When the potential function $V(s_i, s_{-i})$ is rewritten, ith player's potential can be separated as follows.

$$V(s_i, s_{-i}) = \log_2\left(\sum_{\substack{b\in\mathfrak{I},\\ b\neq i}} P_b(\varphi_b, a_b) + P_i(\varphi_i, a_i) + w_0\right) \tag{9}$$

TABLE I
SUBCARRIER AND FSR SELECTION SCHEME

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Game Algorithm
Assign the same subcarriers and carrier frequency shift ratio to each player
for iterations = {1,2,...}
  for i = {1,2,...}
    for $\varphi_i = [0, \ldots, f_0]$
      select an FSR
      for Subcarrier set = {$SS_1$, $SS_2$,...}
        compute $U_i(\varphi_i, a_i)$
      end
    end
    pick/allocate the best resources and FSR to i
  end
end
```

If only the ith player alters its strategy from s_i to s_i', the potential function $V(s_i', s_{-i})$ can be expressed as

$$V(s_i', s_{-i}) = \log_2\left(\sum_{\substack{b\in\mathfrak{I},\\ b\neq i}} P_b(\varphi_b, a_b) + P_i(\varphi_i, a_i)' + w_0\right), \tag{10}$$

When (9) is subtracted from (10), and since the only the ith player changes its strategy, i.e., other players strategies remain the same, the following result is observed.

$$V(s_i', s_{-i}) - V(s_i, s_{-i}) > 0 \tag{11}$$

Thus, the condition in (7) would be satisfied with (11). This proves the NE existence.

C. *Convergence to Nash Equilibrium*

As mentioned in Section III-B, ordinal potential games has at least one pure NE. To show the NE convergence in potential games, finite improvement property (FIP) which is a significant feature of the potential games is being utilized. In FIP, $\xi_i = (s_i^0, s_i^1, s_i^2, \cdots)$ is defined as a sequence of path of the strategy set of S_i for player i. It is assumed that the new and previous strategies of player i are s_i^l and s_i^{l-1}, respectively. Based on the improvement path of ξ_i, the potential function $V(s_i^l)$ should satisfy $V(s_i^0) < V(s_i^1) < V(s_i^2) < \cdots$. Since the players play with their best responses in this letter, the potential function with the new strategy will become $V(s_i^l) > V(s_i^{l-1})$. Therefore, the improvement path of the potential functions would be satisfied. This proves the convergence to unique NE with our algorithm for a given scenario.

IV. PERFORMANCE EVALUATION

We consider there are 20 SBSs which are randomly distributed in a given area, and each SBS serves to 2 SUs. We assume that there are 64 available subcarriers and each SU needs 16 subcarrier. To perform orthogonal transmission with FMTs, we utilize the band limited root raised cosine filter (RRCF) with a roll-off factor β taken as 0.35. We adopt a path loss model which is defined as $PL(\text{dB}) =$

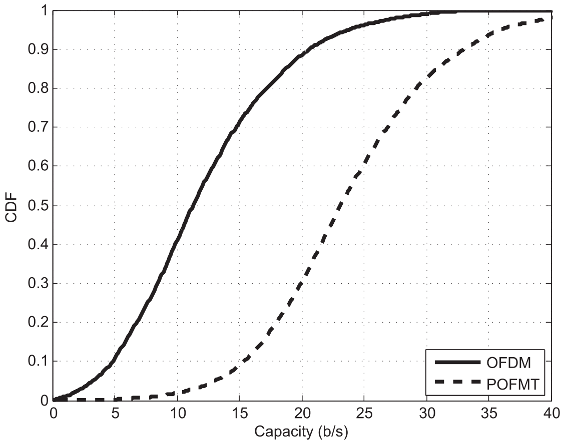

Fig. 3. The resource allocation with POFMT outperforms the resource allocation with OFDM.

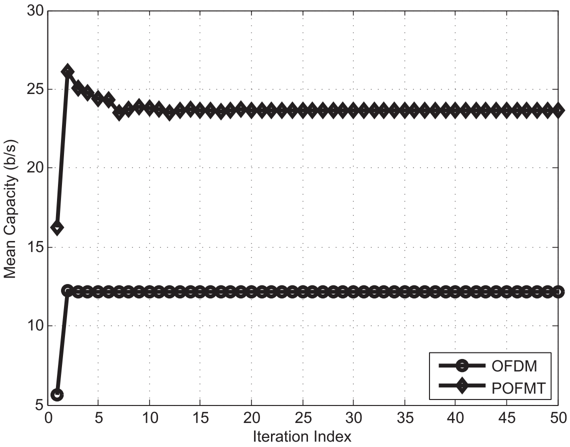

Fig. 4. POFMT algorithm reaches the NE. However, the convergence rate is slow.

$43.3log10(R) + 11.5 + 20 * log10(f_c)$, where R is the distance between transceivers and f_c is the center frequency, which is 2GHz [12]. Finally, the subcarrier spacing is taken as 1.2, i.e., $f_0 = 1 + 0.2$ which introduces slight loss in spectral efficiency.

Figure 3 shows the cumulative distribution function curves of the proposed and existing schemes. As an existing scheme, OFDM is utilized. As seen in figure, the proposed scheme provides higher performance gain in terms of the capacity. While the introducing only CFS decreases the other user interference as shown in [2], CFS with subcarrier allocation provides further reduction in interference. It is basically because each user may select the different subcarriers and/or only some of the subcarriers might be allocated by other user(s). In the mean capacity level, the proposed scheme gives 94% higher capacity gain.

In Fig. 4, the mean capacity of the system in each iteration is depicted for both schemes. This figure proves that the proposed scheme reaches the NE with slightly slower convergence rate which is basically due to the consecutiveness constraint we introduced in our scheme. That is, while some resources might have high capacity gain in the selected subcarriers, some other resources might have lower gain. Therefore, finding the more resources with the highest gain takes longer duration for each user, and hence, equilibrium is achieved more slowly.

V. CONCLUSION

In cognitive HetNets, the users are affected by the high interference coming from the other users in the environment. To mitigate this interference, in this letter, we proposed the game theoretical resource allocation with POFMT in cognitive HetNets. While SBSs look for the consecutive subcarriers which have the highest utility, they also perform the CFS. By jointly selecting the subcarriers and FSR, the system performance was significantly increased in terms of the capacity gain with the expense of slight loss in spectral efficiency. It is noted that the POFMT concept is offered for cognitive HetNets where significant other user interference is possessed. On the other hand, asynchronous transmission is also allowed with POFMT. Due to the constraint we introduced, the convergence rate to NE becomes slower when compared to OFDM. While this concept can be implemented in 4G cognitive HetNets, this can also be considered for the future generations' cognitive HetNets. As a further study, one important contribution can be provided by investigating the optimum gap between subcarriers to figure out the maximum loss in spectral efficiency as opposed to decreasing interference in the system.

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