# OFDM With Subcarrier Number Modulation

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***Abstract*—A new modulation technique, named orthogonal frequency division multiplexing (OFDM) with subcarrier number modulation, is proposed for efficient data transmission. In this scheme, the information bits are conveyed by changing the number of active subcarriers in each OFDM subblock. The idea behind this scheme is inspired from the integration of OFDM with pulse width modulation, where the width of the pulse represents the number of active subcarriers corresponding to specific information bits. This is different from OFDM with index modulation (OFDM-IM), where the information bits are sent by the indices of the subcarriers instead of their number. The scheme is shown to provide better spectral efficiency than that of OFDM-IM at comparable bit error rate performances. Another key merit of the proposed scheme over OFDM-IM is that the active subcarriers can be located in any position within the subblock, thus enabling channel-dependent optimal subcarrier selection that can further enhance the system performance.**

***Index Terms*—OFDM, subcarrier number modulation, index modulation, spectral efficiency, channel-dependent subcarrier selection.**

## I. Introduction

RECENT research studies exhibit the urgent need for designing new advanced waveforms and modulation techniques that are capable of further enhancing spectral efficiency, energy efficiency, and reliability with minimal complexity compared to conventional OFDM in order to fit the diverse needs of future 5G scenarios and services [1], [2].

A novel modulation scheme called spatial modulation that exploits the spatial domain by selecting the indices of antennas along with the classical signal constellations (amplitude/phase modulation) to convey information was proposed in [3]. An improved spatial modulation scheme, whose spectral efficiency *linearly* increases with the transmit antennas' number rather than a base-two logarithm factor, was introduced in [4]. The interesting application of spatial modulation to OFDM system was proposed under the name OFDM with Subcarrier Index Modulation (OFDM-SIM) in [5]. In this scheme, OFDM active subcarriers' indices vary in each OFDM block to convey information bits. A systematic subblock-based transceiver structure, named as OFDM with Index Modulation (OFDM-IM), that enables selecting more than one active subcarrier among the available subcarriers in each subblock was proposed in [6]. In [7], a generalized version of OFDM-IM named as OFDM-GIM was introduced, where different activation ratios per each subblock are used to enhance spectral efficiency. Recently, a comprehensive survey of the recent advances and different variations of index modulation concept was given in [2]. One can describe and perceive the concept of OFDM-IM [6] as OFDM combined with Pulse Position Modulation (PPM), where part of the data bits are conveyed by the positions (indices) of the active subcarriers that can be selected by a simple look-up table. In this approach, a certain fixed number of subcarriers are selected in each subblock as active subcarriers in OFDM-IM to convey data bits.

Different from OFDM-IM, in this letter, we propose a new modulation technique, named as OFDM with Subcarrier Number Modulation (OFDM-SNM), which basically integrates OFDM with Pulse Width Modulation (PWM).[1] The concept of OFDM-SNM proposed in this letter is inspired by PWM by means of representing pulse width by the number of active subcarriers. This number is determined depending on the different combinations of incoming bits. This concept results in creating a new dimension, i.e., number of active subcarriers, for sending data in addition to the conventional two dimensional complex signal constellation plan. These dimensions are jointly utilized to convey information to the receiver by the number of active subcarriers (instead of their indices) alongside the information sent through symbols. The proposed OFDM-SNM results in better spectral efficiency compared to both OFDM and OFDM-IM when BPSK is used. Besides, the power efficiency and reliability of the proposed OFDM-SNM scheme are shown to be better than that of OFDM, and comparable to that of OFDM-IM. Similar to OFDM-IM, the proposed OFDM-SNM has also the potential to reduce Inter-Carrier-Interference (ICI) and Peak-to-Average Power Ratio (PAPR) due to not activating all the subcarriers. Different from OFDM-IM, the activated subcarriers can be placed in any index or position within each subblock as the information is sent by the subcarriers' number, and thus they can be made channel-dependent, resulting in even much better reliability performance. The exact spectral efficiency and error performance formulas of the proposed scheme are derived and shown to be closely matched with the simulated results.

The rest of this letter is organized as follows. The proposed OFDM-SNM is illustrated in Section II. Performance analysis of OFDM-SNM scheme is presented in Section III. Simulation results are carried out in Section IV. Finally, conclusion and future works are provided in Section V.[2]

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[1] Pulse width modulation can also be found in the literature under different names such as pulse interval modulation or pulse duration modulation.

[2] *Notation*: Bold, lowercase and capital letters are used for column vectors and matrices, respectively. $(.)^T$ and $(.)^H$ represent transposition and Hermitian transposition, respectively. $\det(\mathbf{A})$ denotes the determinant of $\mathbf{A}$. $E(.)$ represents the expectation. $\circledast$ denotes a circular convolution operation.

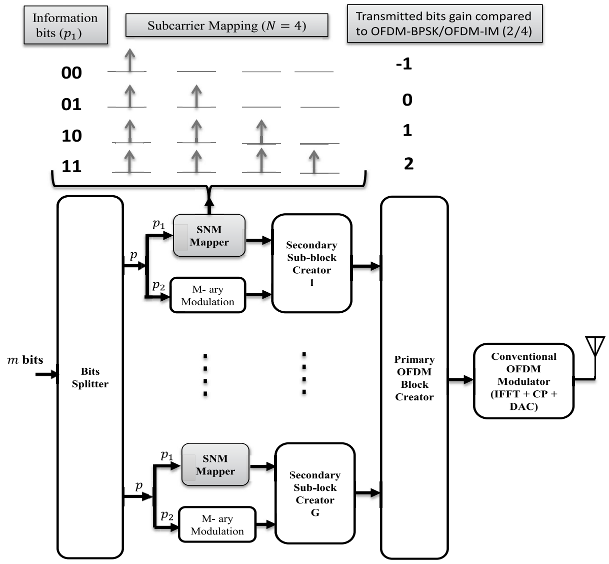


Fig. 1. OFDM-SNM transmitter structure.

TABLE I
SNM Mapper With $p_1$=2 Bits and $N$=4

| Information bits | Active subcarriers |
|---|---|
| [0 0] | [1 0 0 0] |
| [0 1] | [1 1 0 0] |
| [1 0] | [1 1 1 0] |
| [1 1] | [1 1 1 1] |

## II. Proposed OFDM-SNM

The transmitter structure of the proposed OFDM-SNM system is depicted in Fig. 1. For the transmission of each $i_{th}$ OFDM block, a variable number of $m_i$ information bits enter the transmitter of the OFDM-SNM scheme. These bits are split into $G$ groups each containing variable $p = p_1 + p_2$ bits, which are used to form OFDM subblocks of length $N = N_F/G$, where $N_F$ is the size of the Fast Fourier Transform (FFT). Unlike OFDM-IM, where **constant** $K$ out of $N$ number of available subcarriers are used to send information, in our scheme, for each subblock $g$ $(g = 1, 2, \ldots, G)$, **index-independent variable** $K \in [1, 2, \ldots, N]$ out of $N$ available subcarriers are activated by a subcarrier number mapper (selector) according to the corresponding $p_1 = \log_2(N)$ bits, while the remaining N–K subcarriers are inactive. Note that $K$ is not fixed, but rather changing and taking variable values of the number of subcarriers according to the incoming information bits $p_1$ as shown in Table I, which presents the case when $N = 4$, $K \in [1, 2, 3, 4]$ and $p_1 = \log_2(4) = 2$. For each subblock $g$, the subcarrier on-off activation pattern set can be given as

$$\mathbf{i}_g = \begin{bmatrix} i_1 & i_2 & \cdots & i_K \end{bmatrix}^T, \; g = 1, 2, \ldots, G \tag{1}$$

where $i_k \in 1{,}0$ for $k = 1, 2, \ldots, K$. This subcarrier on-off activation pattern procedure can be performed using a look-up table for smaller $N$ and $K$ values as shown in Table I. For each subblock, the remaining $p_2 = K(\log_2(M))$ bits (which change from one subblock to another based on the number of active subcarriers) of the $p$-bit input bit sequence are mapped onto the M-ary signal constellation in order to determine the data symbols that are transmitted over the active subcarriers. After the concatenation of $G$ subblocks, the whole OFDM block can be represented as $\mathbf{x_F} = \begin{bmatrix} x_F(1) & x_F(2) & \ldots & x_F(N_F) \end{bmatrix}$. Now, the remaining steps are performed as done in standard OFDM modulation. After taking IFFT, the signal can be represented as $\mathbf{x_t} = IFFT(\mathbf{x_F})$. By appending CP of length $N_{CP}$ to the transmitted signal, the resulting signal becomes $\mathbf{x_{CP}} = \begin{bmatrix} \mathbf{x_t}(N_F - N_{CP} + 1{:}N_F) & \mathbf{x_t} \end{bmatrix}$. Now, since we assume AWGN multi-path channel with channel impulse response ($\mathbf{h_t}$), the received time-domain signal over the channel can be written as

$$\mathbf{y_t} = \mathbf{x_t} \circledast \mathbf{h_t} + \mathbf{n_t}, \tag{2}$$

where $\mathbf{n_t} \sim \mathcal{CN}(0, N_{o,T})$ is the AWGN vector. The operation of the receiver would be the reversal of the transmitter operations, i.e., removing CP, performing FFT and SNM demapping and detection as follows. The received signal after removing the CP is given by $\mathbf{y_t} = \begin{bmatrix} y_0 & y_1 & \ldots & y_{N_F-1} \end{bmatrix}$. After FFT block, $\mathbf{y_F} = FFT(\mathbf{y_t}) = \begin{bmatrix} y_F(0) & y_F(1) & \ldots & y_F(N_F-1) \end{bmatrix}$. Now, to compensate for the frequency selectivity of the channel, a simple one tap frequency domain equalizer is used and its output can be represented as $\mathbf{y_{eq}} = \mathbf{y_F}/\mathbf{h_F}$. Then, a simple, special energy-based detector is used to extract the active subcarriers pattern using a properly selected threshold value [8]. It is noted that the use of a threshold-based detector facilitates low-complexity receiver for the OFDM-SNM scheme. This detector is much simpler than ML or LLR based detectors [9]. After that, the set of active subcarriers is determined and then mapped to its corresponding bits using SNM demapper, which is the inversion mapping process used at the transmitter. Now, the active subcarriers obtained at the receiver for each subblock is used for constellation symbols detection. Finally, the bits obtained from both SNM demapper and symbol detection are combined to form the final estimated subblock bits. By doing the same procedure for all subblock, the retrieved data stream is obtained for the whole OFDM block.

## III. Performance Analysis of OFDM-SNM

To analyze the performance of the proposed OFDM-SNM, spectral efficiency, pairwise error probability, and power efficiency are derived and investigated as follows.

### A. Spectral Efficiency (SE)

The SE (bits/s/Hz) of the proposed OFDM-SNM scheme can precisely be formulated as

$$\eta_{OFDM-SNM} = \frac{\sum_{g=1}^{G}(\log_2(N) + K(g)\log_2(M))}{N_F + N_{CP}}, \tag{3}$$

where $K$ $(g)$ is the number of active subcarrier in each subblock of $N$ subcarriers' length. It can be observed that the difference between OFDM-SNM and OFDM-IM is in $K(g)$ parameter. In OFDM-SNM, $K(g)$ varies for each subblock, which is different from OFDM-IM where $K(g)$ is fixed for all subblocks. Thus, the proposed OFDM-SNM improves SE over OFDM-IM as well as conventional OFDM. The amount of improvement over OFDM-IM depends on $K(g)$ values. For instance, if $N = 4$ is selected for both OFDM-SNM and OFDM-IM (with $K = 2$), then the SE gain of OFDM-SNM scheme over OFDM-IM, when BPSK is considered, equals to $\rho$ =18/16 = 1.125.

### B. Analytical Bit Error Probability

The activated subcarriers in each sub-block are determined based on the incoming information bits using a mapping process that can basically be represented by a certain code. The reason for this is that there is no information about the exact modulation used in signal constellation, and the known information is that the mapping of the bit information into the number of active subcarriers. Assume there are $T$ time slots which represent the number of bit group, then the transmitted codeword in $T$ time slots $\mathbf{X} = \mathbf{X_{ij}}$, where $i = 1, 2, \ldots, T$ and $j = 1, 2, \ldots, N$. At the receiver, the decoder may decode another codeword $\hat{\mathbf{X}} = \hat{\mathbf{X}}_{\mathbf{ij}}$. Our system model has just those two codewords ($\mathbf{X}$ and $\hat{\mathbf{X}}$), so here the analytical BER considers only pairwise error probability ($P(\mathbf{X} \longrightarrow \hat{\mathbf{X}})$). It should be noted that the receiver might cause errors on the two consecutive detection processes, i.e., the number of active subcarriers as well as the $M$-ary symbols. In the considered model, we assume that the frequency selective channel is fixed within one block and follows Rayleigh distribution. The input-output relationship in the frequency domain can be written in the following form

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{n_z}. \tag{4}$$

The transmitted sequence $\mathbf{X}$ could be detected correctly or erroneously as $\hat{\mathbf{X}}$. An optimal detector for the proposed system can be represented mathematically as

$$P(\mathbf{X} = \mathbf{X}|\mathbf{y}) \geq P(\mathbf{X} = \hat{\mathbf{X}}|\mathbf{y}). \tag{5}$$

We assume that $\mathbf{n_z} \sim \mathcal{CN}(0, N_{o,F})$, then the detector is simply ML detector defined as

$$|\mathbf{y} - \mathbf{X}| < |\mathbf{y} - \hat{\mathbf{X}}|. \tag{6}$$

The distance from $\mathbf{y}$ to $\mathbf{X}$ is less than that to $\hat{\mathbf{X}}$, so we can say that $\mathbf{X}$ is the nearest neighbor to $\mathbf{y}$. Since we have only two probabilities then the threshold value is assumed to be at the middle point between $\mathbf{X}$ and $\hat{\mathbf{X}}$ as $\frac{\mathbf{X}+\hat{\mathbf{X}}}{2}$. So, the probability of choosing $\mathbf{X}$ can be computed as

$$\begin{aligned} P(\mathbf{y} < \frac{\mathbf{X}+\hat{\mathbf{X}}}{2}|\mathbf{X} = \mathbf{X}) &= P(\mathbf{z} > \frac{||\mathbf{X}-\hat{\mathbf{X}}||}{2}) \\ &= Q(\frac{||\mathbf{X}-\hat{\mathbf{X}}||}{2\sqrt{N_o/2}}), \end{aligned} \tag{7}$$

where $Q(.)$ is the Q-function [10] and $N_o = N_{o,F}$.

Accordingly, the error probability depends only on the distance between the sequences: $\mathbf{X}$ and $\hat{\mathbf{X}}$; with the effect of the channel ($\mathbf{h}$), the Q function becomes [11]

$$\begin{aligned} P(\mathbf{y} < \frac{\mathbf{X}+\hat{\mathbf{X}}}{2}|\mathbf{X} = \mathbf{X}) &= Q(\sqrt{\frac{||(\mathbf{X}-\hat{\mathbf{X}})\mathbf{h}||^2}{2N_o}}) \\ &= Q(\sqrt{\frac{\mathbf{h^H}(\mathbf{X}-\hat{\mathbf{X}})^H(\mathbf{X}-\hat{\mathbf{X}})\mathbf{h}}{2N_o}}). \end{aligned} \tag{8}$$

Equation (8) can be rewritten as

$$P(\mathbf{y} < \frac{\mathbf{X}+\hat{\mathbf{X}}}{2}|\mathbf{X} = \mathbf{X}) = Q(\sqrt{\frac{\mathbf{h^H}\mathbf{A}\mathbf{h}}{2N_o}}) = Q(\sqrt{\frac{\delta}{2N_o}}), \tag{9}$$

where $\delta = \mathbf{h^H}\mathbf{A}\mathbf{h} = \mathbf{h^H}(\mathbf{X}-\hat{\mathbf{X}})^H(\mathbf{X}-\hat{\mathbf{X}})\mathbf{h} = ||(\mathbf{X}-\hat{\mathbf{X}})\mathbf{h}||^2$, and the $\mathbf{A}$ matrix equals to $(\mathbf{X}-\hat{\mathbf{X}})^H(\mathbf{X}-\hat{\mathbf{X}})$. The channel frequency response $\mathbf{h_F}$ is assumed to be zero-mean Circularly Symmetric Complex Gaussian (CSCG) random variable with unity variance $\mathbf{h_F} \sim \mathcal{CN}(0, 1)$. $\mathbf{h_F}$ can be completely described by its mean and covariance matrix $\mathbf{K} = E[\mathbf{h_F}\mathbf{h_F}^H]$, where $\mathbf{K}$ is a Hermiation matrix with a dimension of $N < N_F$. So, a submatrix $\mathbf{K_N}$ with size $N \times N$ is sufficient to represent the channel frequency response in each subblock. If we define a complex orthogonal matrix $\mathbf{Q}$ where $\mathbf{Q^H}\mathbf{Q} = \mathbf{I}$, $\mathbf{h}$ as a subset of $\mathbf{h_F}$ can be represented as orthonormal basis $\mathbf{h} = \mathbf{Q}\mathbf{u}$, and $\mathbf{D}$ as a diagonal matrix with rank $r_1 < N$, where $\mathbf{D} = E[\mathbf{u}\mathbf{u^H}] = E[\mathbf{h}\mathbf{h^H}]$, and $\mathbf{K_N}$ can be given as

$$\mathbf{K_N} = E[\mathbf{h}\mathbf{h^H}] = \mathbf{Q}\mathbf{D}\mathbf{Q^H}. \tag{10}$$

The PDF of the orthonormal basis $\mathbf{u}$ follows the PDF of $\mathbf{h}$ and can be expressed as

$$f(\mathbf{u}) = \frac{exp(-\mathbf{u^H}\mathbf{D}^{-1}\mathbf{u})}{\pi^{r1}det(\mathbf{D})}. \tag{11}$$

The Conditional Pairwise Error Probability (CPEP) is the same as equation (9) and the PDF of the fading channel is represented in equation (11). Then, by taking the expectation for CPEP with respect to the channel, and using the approximation of Q function found in [10], the Unconditional Pairwise Error Probability (UPEP) can be obtained as

$$P(\mathbf{X} \longrightarrow \hat{\mathbf{X}}) = \frac{1/12}{det(\mathbf{I_N} + q_1\mathbf{K_N}\mathbf{A})} + \frac{1/4}{det(\mathbf{I_N} + q_2\mathbf{K_N}\mathbf{A})}, \tag{12}$$

where $q_1 = 1/(4N_{o,F})$ and $q_2 = 1/(3N_{o,F})$. The overall bit error probability is of great interest rather than individual PEP. The Average Bit Error Probability (ABEP) can be calculated as follows [6]:

$$P_b(E) \approx \frac{1}{p\ n_x}\sum_{\mathbf{X}}\sum_{\hat{\mathbf{X}}} P(\mathbf{X} \longrightarrow \hat{\mathbf{X}})\ e(\mathbf{X}, \hat{\mathbf{X}}), \tag{13}$$

where $p$ is the number of information bits per subblock transmission, $n_x$ represents the number of realizations of $\mathbf{X}$, and $e(\mathbf{X}, \hat{\mathbf{X}})$ is the number of information bit errors committed by choosing $\hat{\mathbf{X}}$ instead of $\mathbf{X}$. It is shown analytically that the BER of OFDM-SNM is similar to that of OFDM-IM [6]. This analytical result will be verified by simulation as well.

### C. Power Efficiency

Since not all subcarriers are occupied where only active subcarriers carry modulated data, OFDM-SNM approach achieves better power efficiency compared to conventional OFDM. In conventional OFDM, the transmitted power ($P_{tx}$) is distributed equally among all subcarriers so that the average power per subcarrier equals to ($P_{tx}/N_F$). The distribution of observing the number of active subcarriers in each subblock is assumed to follow binomial distribution as

$$P(N_{ac} = K) = \binom{N}{K} p_r^K (1-p_r)^{N-K}, \tag{14}$$

where $N_{ac}$ represents the number of active subcarriers which takes values of $K = 1, 2, \ldots, N$. Also, $p_r$ is the probability of the event ($N_{ac} = K$). By assuming that the total transmitted

TABLE II
SIMULATION PARAMETERS

| | |
|---|---|
| Modulation type | BPSK ($M = 2$) |
| IFFT/FFT size ($N_F$) | 64 |
| CP Guard Interval (samples) | 8 |
| Number of subblocks in each OFDM symbol ($G$) | 16 |
| Number of available subcarriers in each subblock ($N$) | 4 |
| Number of bits mapped to each subblock ($p_1$) | 2 |
| Multipath channel delay samples locations | [0 3 5 6 8] |
| Multipath channel tap power profile (dBm) | [0 -8 -17 -21 -25] |

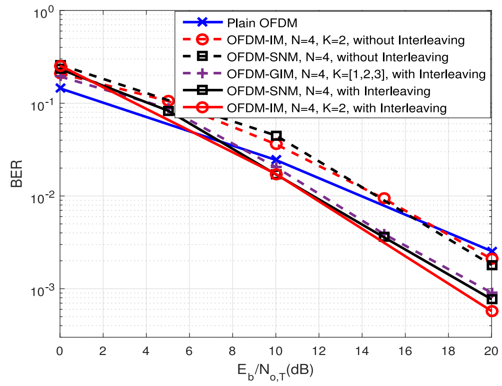


Fig. 2. BER of OFDM-SNM compared to conventional schemes.

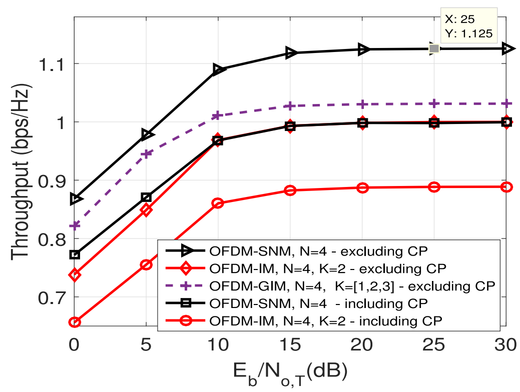


Fig. 3. Throughput of OFDM-SNM compared to conventional schemes.

power allocated to OFDM-SNM block is $P_{tx}$, and the power is equally distributed to all subblocks, the power consumed by OFDM-SNM scheme can be written as

$$P_c = \frac{P_{tx}}{G\ N} \sum_{g=1}^{G} K(g)\ P(N_{ac} = K(g)), \tag{15}$$

where $K(g)$ and $P(N_{ac} = K(g))$ represent the number of active subcarriers and their corresponding probability in the subblock $g$, respectively. Thus, the average power allocated per subcarrier equals to $P_c/N_F$.

## IV. SIMULATION RESULTS

In this section, BER and throughput performances of OFDM-SNM system are simulated and compared with both standard OFDM and OFDM-IM. The simulation parameters used are shown in Table II. It is assumed that the multi-path channel is Rayleigh distributed. Fig. 2 shows BER vs. $E_b/N_{o,T}$ of the proposed OFDM-SNM system compared to its competitive systems such as conventional OFDM, OFDM-IM [6] (with $K = 2$ and $N = 4$) and OFDM-GIM [7] (with $N = 4$ and $K = [1, 2, 3]$). As seen from Fig. 2, both OFDM-SNM and OFDM-IM have similar BER performance which is better than that of classical OFDM for high SNR values (SNR $> 10$ *dB*). Also, it is observed that the BER can be improved further when interleaving is adopted [12]. Fig. 3 demonstrates that the throughput performance of the proposed OFDM-SNM (in both cases when CP is included and excluded from the throughput calculation) is better than that of OFDM-IM by a factor of 1.125 and than that of OFDM-GIM by a factor of 1.1 at equivalent BER performances.

## V. CONCLUSION

This letter introduces a novel multi-carrier modulation scheme called OFDM-SNM that sends information not only by symbols but also by the number (instead of indices) of active subcarriers in each subblock. OFDM-SNM improves the spectral efficiency compared to that of OFDM-IM by 12.5%. Besides, different from OFDM-IM, since the active subcarriers in OFDM-SNM send information by their number instead of indices, their mapping can be configured to be floating or contiguous based on the channel quality or the ICI level between subblocks, resulting in even a better performance. Validating these extra merits alongside investigating OFDM-SNM with different modulation orders and subblock sizes will be key subjects of future research studies.

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