

# A Novel One-Base Station Hybrid Positioning Method

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**Abstract**—In wireless networks, the need for accurate and low complexity localization methods are growing. Although many positioning methods based on signals' time difference of arrival (TDOA) and angle of arrival (AOA) have been proposed, these methods require multiple base stations (BS) and calculations with high complexity. Furthermore, the distance between a target and the BSs are usually larger than the distance between different BSs, which causes geometric dilution of precision (GDP) problem. To circumvent these issues, we propose a novel and linear method for positioning by only one BS. Our method uses both AOA and TDOA of incoming signals and called "positioning using one BS (PuOB)". In this method, we take measurements from one BS at different time instances instead of taking measurements from multiple BSs simultaneously. The ability to estimate the mobile transmitter position accurately by using one BS is the highlighted advantage of the PuOB over the conventional methods. The positioning accuracy of PuOB for different BS numbers are presented. According to simulation results, PuOB outperforms TDOA and AOA methods using three and two BSs, respectively.

**Index Terms**—Positioning, base station (BS), time difference of arrival (TDOA), angle of arrival (AOA).

## I. INTRODUCTION

Positioning and tracking of mobile transmitters are important parts of new wireless navigation systems and cellular radio networks. A positioning method requires to satisfy the imposed requirements, such as high reliability and accuracy along with keeping the costs and complexity low. Methods based on time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA) and received signal strength (RSS) of the signals [1]- [3] are the most significant and well-known approaches.

The TOA method requires knowledge of the time instances that a signal is transmitted and received; on the other hand, TDOA requires only the time difference of arrivals at different receivers. In this study, we mainly focus on cellular networks thus Base Stations (BS) are considered as receivers. It is worth mentioning that both TOA and TDOA methods need an accurate synchronization between different BSs which increases complexity of the methods. In [4] and [5], authors removed the need of synchronization between BSs by considering the location of mobile transmitter in two different time instances, while requiring the utilization of multiple BSs for the estimation of the location of the transmitter. The AOA method unlike the TOA and TDOA methods uses angles of received signal in different BSs. For AOA measurements, either directional antennas or antenna arrays at each BS are required [6]. Although AOA have some advantages over TDOA, it is not

precise in non-line of sight (NLOS) scenarios thus TDOA is preferred more by systems designers [6]. RSS method is another method utilizing the known power of transmitted signal for positioning; however, its precision is less than the previous methods due to multipath fading [7].

For the mentioned methods number of BSs and cost are significant issues. The minimum number of required BSs that AOA and TDOA methods require for 3-dimensional (3-D) positioning are two and four BSs, respectively.

Recently, in [8]- [10] new methods are introduced, requiring at least two BSs to have an accurate estimation. The methods are inspired from previous positioning methods like AOA and TDOA. In [8], the authors proposed a method in which direction of the transmitter is estimated using TDOA while AOA method is used to have the initial range. Although the method given in [8] proposes a linear hybrid method and solves the non-convergence problem of TDOA utilizing AOA method, it does not consider number of BSs and cost. In [9], authors proposed a combined TDOA-AOA positioning (TAP) method. Measurements from AOA and TDOA and the unknown position of the transmitter are used to construct mathematical relations in order to find the unknown position. The performance of this method highly depends on AOA measurements. In [10], an approach based on AOA is proposed considering Doppler shift for positioning of a mobile source. Instead of estimating at the BSs, the unknown position is estimated at the mobile target in [10]. The method requires LOS transmission like AOA method with using two BSs in 2-D coordinates. Consequently, it is necessary to have more BSs for 3D-positioning estimations.

In addition, most of the mentioned methods suffer from a problem called geometric dilution precision (GDP) happens when the distance of transmitter from BSs is more than distances between BSs. It causes to lose the precision in estimations or even make the position finding equations non-solvable [8]. Although some of the methods like the method in [8] try to decrease GDP problem, it increases the complexity and cost.

In this study, a novel positioning method is proposed to solve the mentioned restrictions in order to estimate the location of a distant mobile transmitter based on unknown received signal power at only one BS. Thus, there is no necessity of synchronization between different BSs. In addition, we propose a highly reliable positioning method called **positioning using one BS (PuOB)**. The new method is based on AOA and TDOA

measurements which are taken at one BS in different time instances. The contributions of this work are:(I) Introduction of the PuOB method, a new positioning method using only one BS,(II) Comparison of the PuOB method with AOA and TDOA and TAP methods in realistic scenarios.

The rest of the paper is organized as follows. System model of the new method along with TDOA and AOA is described in Section II. In Section III, simulations and comparisons are provided. Finally, conclusion is given in Section IV.

## II. SYSTEM DESCRIPTION

The PuOB method utilizes AOA and TDOA positioning methods in order to achieve a highly reliable and low cost hybrid positioning estimation and tracking method.

The first and second parts of the system description is dedicated to TDOA and AOA equations used in this work. The third part describes the PuOB method.

**Notation:**  $M$  and  $N$  are the total number of assumed BSs in an arbitrary positioning method and total number of time instances that the measurements are taken, respectively.  $\|\cdot\|$  and  $(\cdot)^T$  denote the  $L^2$ -norm operation and the transpose operation of a matrix, respectively. Let  $c$  denote the velocity of the transmitted signal.

### A. Time Difference of Arrival (TDOA) Method

The conventional TDOA method uses the time of arrival differences between a received signal at different BSs. As shown in Fig. 1, each BS acts as a focus of a hyperbola in conventional TDOA method. The intersection of these hyperbolas provides the estimated location of the transmitter.

Let  $\mathbf{S}_m$  denote  $m^{\text{th}}$  BS located at  $[x_m, y_m, z_m]^T \in \mathbb{R}^3$  in Cartesian coordinates where  $m = 1, 2, \dots, M$ . The coordinates of the mobile transmitter at the time of  $t_n$ ,  $n = 0, 1, \dots, N$  is defined as  $\mathbf{P}_n = [x_n, y_n, z_n]^T \in \mathbb{R}^3$ . Then, in the absence of measurement noise, distance  $r_{m,n}$  between the transmitter at time instance of  $t_n$  and  $m^{\text{th}}$  BS can be modeled as

$$\begin{aligned} r_{m,n} &= cT_{m,n} = \|\mathbf{P}_n - \mathbf{S}_m\| \\ &= \sqrt{(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2}, \end{aligned} \quad (1)$$

where  $T_{m,n}$  is the duration for the signal to travel from the transmitter to the BS. By considering two BSs as foci of a hyperbola, the distance difference between the transmitter and the two BSs,  $m^{\text{th}}$  and  $(m-1)^{\text{th}}$  BSs, can be given as  $d_{(m,m-1),n} = c(T_{m,n} - T_{(m-1),n}) = r_{m,n} - r_{(m-1),n}$ , where  $d_{(m,m-1),n}$  is fixed-difference property that can be used for positioning on hyperbolas ( $|r_{m,n} - r_{(m-1),n}| = d_{(m,m-1),n}$  for every point on the hyperbolas). As shown in Fig. 1, for a simple hyperbola, at least two BSs are required. In order to have a TDOA positioning estimation of a transmitter in 3-D coordinates, at least 3 hyperbolas produced by measurements at 4 BSs are necessary.

### B. Angle of Arrival (AOA) Method

In this part, we extract the relations of AOA which is used in the PuOB method.  $\mathbf{S}_m$  is assumed to be in the origin of Cartesian coordinates, in  $r_{n,m}$  distance of  $\mathbf{P}_n$  as given in (2).

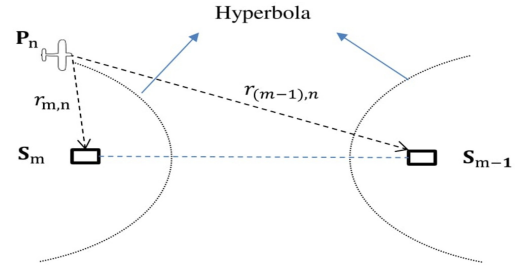


Fig. 1: Hyperbola made by TDOA measurements in two BSs located at  $\mathbf{S}_m$  and  $\mathbf{S}_{m-1}$  as foci and the target located at  $\mathbf{P}_n$ .

The unit vector from BS toward the transmitter is the resultant of three vectors in each Cartesian axis,

$$\mathbf{P}_n - \mathbf{S}_m = r_{m,n} \begin{bmatrix} \cos \alpha_{m,n} & \cos \beta_{m,n} & \cos \gamma_{m,n} \end{bmatrix}^T, \quad (3)$$

where  $\alpha_{m,n}$ ,  $\beta_{m,n}$  and  $\gamma_{m,n}$  are the angles of received signal with x, y, z axes. In order to measure the angles of arrival at each axis, similar to [11], we assume antenna arrays are used. As shown in Fig. 2, for measuring  $\alpha_{m,n}$  which is the angle of received signal with x-axis at  $m^{\text{th}}$  BS, there is a simple antenna array in x-direction. So,  $\alpha_{m,n}$  is defined as  $\alpha_{m,n} = \cos^{-1} \left( \frac{c|t_2 - t_1|}{\ell} \right)$ , where  $t_1$  and  $t_2$  are the time instances of received signal to antenna 1 and antenna 2, respectively.  $\ell$  is the known distance between two antennas.  $\beta_{m,n}$  and  $\gamma_{m,n}$  can be found like  $\alpha_{m,n}$  with a 3D antenna setup.

Equation (3) presents that the vector  $\mathbf{P}_n - \mathbf{S}_m$  is equal to  $r_{m,n}$  multiplied by the unit vector of  $\mathbf{P}_n - \mathbf{S}_m$ . Let  $\mathbf{b}_{m,n}$  denote the unit vector,

$$\mathbf{P}_n - \mathbf{S}_m = r_{m,n} \mathbf{b}_{m,n}. \quad (4)$$

According to (3) and Fig. 3,  $\mathbf{b}_{m,n}$  in Spherical coordinates is defined as:

$$\mathbf{b}_{m,n} = \begin{bmatrix} \sin \theta_{m,n} \cos \phi_{m,n} & \sin \theta_{m,n} \sin \phi_{m,n} & \cos \theta_{m,n} \end{bmatrix}^T \quad (5)$$

In (5),  $\theta_{m,n} \in [0, \pi]$  and  $\phi_{m,n} \in [0, 2\pi]$  are defined as

$$\mathbf{A}_{m,n} = \begin{bmatrix} \phi_{m,n} \\ \theta_{m,n} \end{bmatrix} = \begin{bmatrix} \tan^{-1} \left( \frac{\cos \beta_{m,n}}{\cos \alpha_{m,n}} \right) \\ \tan^{-1} \left( \frac{\sqrt{\cos^2 \alpha_{m,n} + \cos^2 \beta_{m,n}}}{\cos \gamma_{m,n}} \right) \end{bmatrix}, \quad (6)$$

where  $\mathbf{A}_{m,n}$  denotes the AOA measurements matrix.

### C. Positioning using One Base Station (PuOB) Method

Fig. 3 depicts how a BS and position of a transmitter in two time instances can be used by the PuOB method. The transmitter in two time instances plays the role of foci in a hyperbola and the BS is located on hyperbola. In order to find the unknown position of  $\mathbf{P}_n$ , AOA and TDOA measurements are gathered in matrix  $\mathbf{k}_{m,n} \in \mathbb{R}^3$  as  $\mathbf{k}_{m,n} \triangleq \begin{bmatrix} d_{m,(i,n)} & \mathbf{A}_{m,n}^T \end{bmatrix}$ , where  $\mathbf{A}_{m,n}$  is the same as in (8) and  $d_{m,(i,n)}$  can be defined as

$$d_{m,(i,n)} = r_{m,n} - r_{m,i}. \quad (7)$$

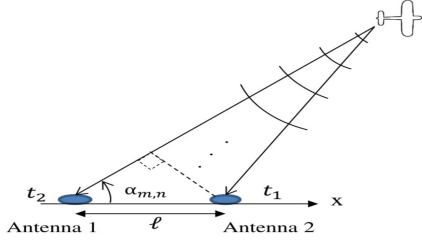


Fig. 2: The x-D antenna array of the  $m^{\text{th}}$  BS for measuring angle of arrival  $\alpha_{m,n}$ .

where  $r_{m,i}$  is the distance of the transmitter at the time instance  $t_i$  ( $i < n$ ) from  $m^{\text{th}}$  BS. It is worth mentioning that  $m$  in this method, the PuOB, is one ( $M = m = 1$ ), because only one BS is required.

The noisy version of  $\mathbf{k}_{m,n}$ , denotes as  $\hat{\mathbf{k}}_{m,n}$  at the  $m^{\text{th}}$  BS is received as

$$\hat{\mathbf{k}}_{m,n} = \mathbf{k}_{m,n} + e, \quad (8)$$

where  $e$  is zero mean additive white Gaussian noise with the standard deviations of TDOA and two AOA measurements equal to  $[\sigma_d \ \sigma_\phi \ \sigma_\theta]^T$ , respectively.

By considering  $m^{\text{th}}$  BS and  $N$  measurement time instances, the measurements at the BS can be defined as  $\mathbf{K} \triangleq [\hat{\mathbf{k}}_{m,1} \ \hat{\mathbf{k}}_{m,2} \ \cdots \ \hat{\mathbf{k}}_{m,N}] = [\hat{d}_{m,(0,1)} \ \hat{\mathbf{A}}_{m,1}^T \ \cdots \ \hat{d}_{m,(N-1,N)} \ \mathbf{A}_{m,N}^T]$ , where matrix  $\mathbf{K} \in \mathbb{R}^N$  is the complete system measurements in  $m^{\text{th}}$  BS and all the time instances.

A matrix  $\mathbf{G} \in \mathbb{R}^{3 \times 2}$  is defined in [9] in a way that its columns are an orthogonal basis of the plane orthogonal to the unit vector of the transmitter position with respect to the corresponding BS. We require a similar  $\mathbf{G}_{m,n}$  to have

$$\mathbf{G}_{m,n}^T \mathbf{b}_{m,n} = 0, \quad (9)$$

where

$$\mathbf{G}_{m,n} = \begin{bmatrix} \sin \varphi_{m,n} & \cos \theta_{n,m} \cos \varphi_{n,m} \\ -\cos \varphi_{m,n} & \cos \theta_{m,n} \sin \varphi_{m,n} \\ 0 & -\sin \theta_{m,n} \end{bmatrix}. \quad (10)$$

Then, by substituting (4) for  $\mathbf{b}_{m,n}$  in (9),

$$\mathbf{G}_{m,n}^T (\mathbf{P}_n - \mathbf{S}_m) = 0. \quad (11)$$

This results in

$$\mathbf{G}_{m,n}^T \mathbf{P}_n = \mathbf{G}_{m,n}^T \mathbf{S}_m. \quad (12)$$

Equation (12) can be written for any time instance, thus for  $t_i^{\text{th}}$  time instance,

$$\mathbf{G}_{m,i}^T \mathbf{P}_i = \mathbf{G}_{m,i}^T \mathbf{S}_m. \quad (13)$$

Adding (12) and (13) for two different time instances results

$$\mathbf{G}_{m,n}^T \mathbf{P}_n + \mathbf{G}_{m,i}^T \mathbf{P}_i = \mathbf{G}_{m,n}^T \mathbf{S}_m + \mathbf{G}_{m,i}^T \mathbf{S}_m. \quad (14)$$

For the time instances of  $t_n$  and  $t_i$  based on orthogonality of both  $(\mathbf{b}_{m,n} - \mathbf{b}_{m,i})$  and  $(\mathbf{b}_{m,n} + \mathbf{b}_{m,i})$  vectors [9] through (5), we have

$$r_{m,n}(\mathbf{b}_{m,n} - \mathbf{b}_{m,i})^T (\mathbf{b}_{m,n} + \mathbf{b}_{m,i}) = 0. \quad (15)$$

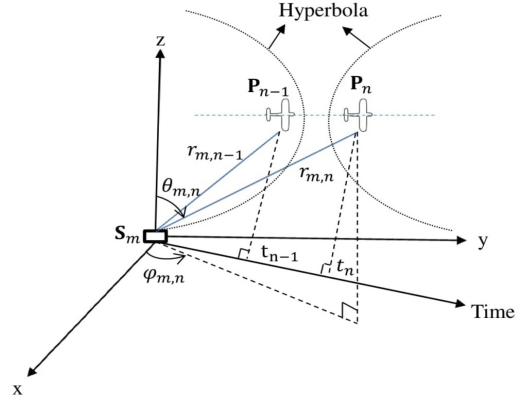


Fig. 3: The PuOB positioning of a target located at  $\mathbf{P}_n$  with one BS located at  $\mathbf{S}_m$ .

By using the expression of  $r_{m,n}(\mathbf{b}_{m,n} + \mathbf{b}_{m,i})$  in (15) and  $r_{m,n} = r_{m,i} + d_{m,(i,n)}$  obtained from (7), we have

$$\begin{aligned} r_{m,n}(\mathbf{b}_{m,n} + \mathbf{b}_{m,i}) &= r_{m,n} \mathbf{b}_{m,n} + r_{m,i} \mathbf{b}_{m,i} \\ &= r_{m,n} \mathbf{b}_{m,n} + r_{m,i} \mathbf{b}_{m,i} + d_{m,(i,n)} \mathbf{b}_{m,i}. \end{aligned} \quad (16)$$

Equation (17) is achieved by substituting  $r_{n,m}(\mathbf{b}_{m,n} + \mathbf{b}_{m,i})$  from (16) into (15)

$$\begin{aligned} &(\mathbf{b}_{m,n} - \mathbf{b}_{m,i})^T [ \underbrace{r_{m,n} \mathbf{b}_{m,n}}_{\mathbf{P}_n - \mathbf{S}_m} + \underbrace{r_{m,i} \mathbf{b}_{m,i}}_{\mathbf{P}_i - \mathbf{S}_m} + d_{m,(i,n)} \mathbf{b}_{m,i} ] \\ &= (\mathbf{b}_{m,n} - \mathbf{b}_{m,i})^T [ \mathbf{P}_n + \mathbf{P}_i - 2\mathbf{S}_m + d_{m,(i,n)} \mathbf{b}_{m,i} ] = 0. \end{aligned} \quad (17)$$

Therefore:

$$\begin{aligned} &(\mathbf{b}_{m,n} - \mathbf{b}_{m,i})^T (\mathbf{P}_n + \mathbf{P}_i) = \\ &(\mathbf{b}_{m,n} - \mathbf{b}_{m,i})^T (2\mathbf{S}_m - d_{m,(i,n)} \mathbf{b}_{m,i}). \end{aligned} \quad (18)$$

Equations (14) and (18) define the necessary equations for PuOB estimation. Estimated multiplier matrix of unknown transmitter position at time instance  $t_n$  is denoted by  $\hat{\mathbf{F}}_{m,n} \in \mathbb{R}^{3 \times 3}$  which can be extracted from (14) and (18)

$$\hat{\mathbf{F}}_{m,n} = \begin{bmatrix} (\hat{\mathbf{b}}_{m,n} - \hat{\mathbf{b}}_{m,i})^T \\ \hat{\mathbf{G}}_{m,n}^T \end{bmatrix}, \quad (19)$$

and similarly  $\hat{\mathbf{F}}_{m,i}$  for known transmitter position at time instance  $t_i$ . Then we will have:

$$\hat{\mathbf{F}}_{m,n} \hat{\mathbf{P}}_n + \hat{\mathbf{F}}_{m,i} \hat{\mathbf{P}}_i = \hat{\mathbf{D}}, \quad (20)$$

where  $\hat{\mathbf{D}} \in \mathbb{R}^3$  using (14) and (18) is obtained as

$$\hat{\mathbf{D}} = \begin{bmatrix} (\hat{\mathbf{b}}_{m,n} - \hat{\mathbf{b}}_{m,i})^T (2\mathbf{S}_m - \hat{d}_{m,(i,n)} \hat{\mathbf{b}}_{m,i}) \\ \hat{\mathbf{G}}_{m,n}^T \mathbf{S}_m + \hat{\mathbf{G}}_{m,i}^T \mathbf{S}_m \end{bmatrix}. \quad (21)$$

Finally, the estimated position of the transmitter can be obtained from:

$$\hat{\mathbf{P}}_n = \hat{\mathbf{F}}_{m,n}^{-1} (\hat{\mathbf{D}} - \hat{\mathbf{F}}_{m,i} \hat{\mathbf{P}}_i). \quad (22)$$

Note that in (19) to (22),  $(\hat{\cdot})$  denotes the estimated values obtained from the measurements.

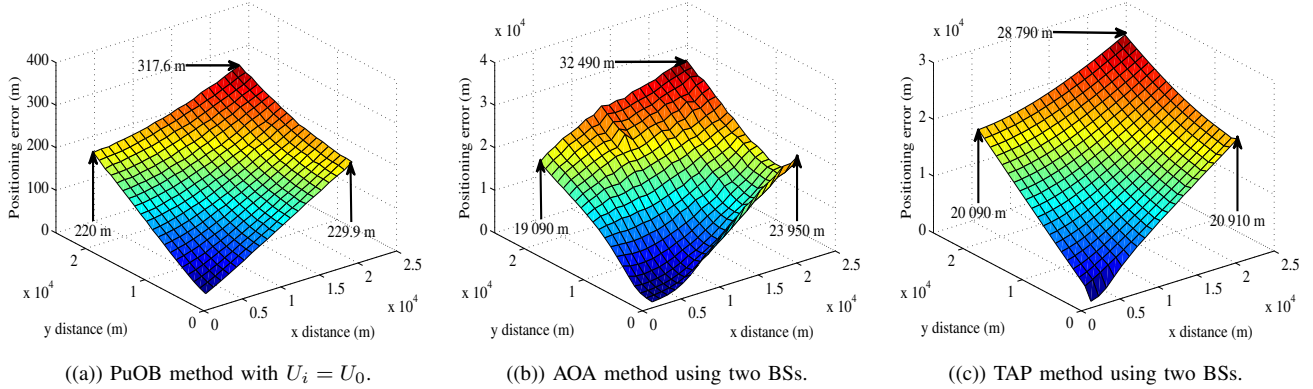


Fig. 4: Average localization error of PuOB, AOA and TAP methods, for different x,y locations of a target. (The height of the target,  $\sigma_{TDOA}$  and  $[\sigma_\phi \ \sigma_\theta]^T$  are set to 1000 m, 30 ns and  $[0.5 \ 0.5]^T$  degrees, respectively.)

### III. PERFORMANCE EVALUATION

In this section, we compare the performance of the PuOB method utilizing one BS with AOA and TAP methods utilizing two BSs in order to have a fair comparison according to the least number of BSs. In AOA and TAP methods the BSs are located in 400 meters apart from each other, the same distance as it is common in urban cellular networks ( $\mathbf{S}_1 = [0 \ 0 \ 0]^T$  and  $\mathbf{S}_2 = [400 \ 0 \ 0]^T$  meters). The BS used in PuOB is located at  $\mathbf{S}_1 = [0 \ 0 \ 0]^T$ . We consider four simulation scenarios. In the first two, the location of the transmitter is varied in order to observe the performance of the methods in terms of distance between transmitter and BSs. During the first simulation scenario, position of the transmitter is varied from  $\mathbf{P}_0 = [1000 \ 1000 \ 1000]^T$  meters till  $\mathbf{P}_N = [20000 \ 20000 \ 1000]^T$  meters.  $\sigma_{TDOA} = \sigma_d/c$  denotes the standard deviation of time difference of arrival measurements and it is assumed to be 30 ns with respect to the distance between BSs in urban cellular networks and  $[\sigma_\phi \ \sigma_\theta]^T$  are set to  $[0.5 \ 0.5]^T$  degrees based on the range of signal diffraction in [9]. Figs. 4(a), (b) and (c) show the average error performance of PuOB, AOA and TAP methods based on different positions of transmitter. The results present that PuOB and TAP methods are more stable than AOA method with respect to the position of transmitter. The AOA method estimation highly depends on the position of transmitter with respect to the position of BSs. It is shown that  $\sigma_{\phi,\theta} = 0.5$  degree in pure AOA method leads to large error in far distances. As shown in Fig. 4(b), the estimation in x-direction, where two BSs are located, is getting worse because of narrowness and similarity of the AOA measurements in both BSs. But in y-direction it has better performance than x-direction because of having different and clear angles in two BSs located in x-axis. Both of the PuOB and TAP methods are more symmetric and stable in different angles due to the nature of using hybrid AOA and TDOA measurements. In fact, TDOA measurements in PuOB and TAP can decrease the dependency on AOA measurements; however, TAP method

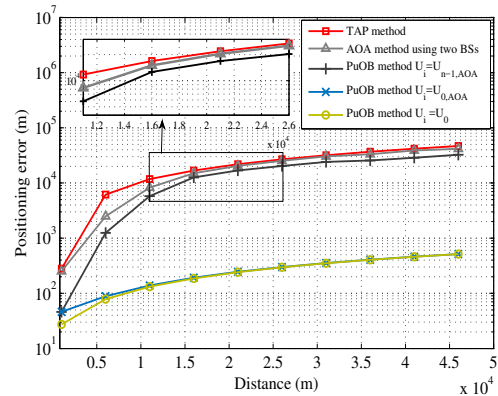


Fig. 5: Performance comparison of PuOB with different initializations, AOA and TAP localizations of a target located in y-axis while BSs are located in x-axis.

utilizes more AOA measurements than PuOB causing more dependency on AOA measurements than PuOB. In addition, in case of GDP problem, PuOB outperforms two other methods. At maximum distance, average error observed by PuOB is 317.6 m, while in AOA and TAP methods, average errors reach up to 32 km and 28 km, respectively.

In the second scenario, the PuOB method is initialized with three different initial positions ( $\mathbf{P}_i$ ). The transmitter's position is varied in y-direction and other parameters such as  $\sigma_{TDOA}$  and  $[\sigma_\phi \ \sigma_\theta]^T$  are the same as the first scenario. In other words, this simulation shows what Fig. 4 provides in y-direction with different initializations in PuOB. Three different initial transmitter's positions are: I The exact position of transmitter at  $t_0$ , ( $\mathbf{P}_i = \mathbf{P}_0$ ), II The estimated position of transmitter by AOA method at  $t_0$  ( $\mathbf{P}_i = \mathbf{P}_{0, AOA}$ ), III The estimated position of transmitter by AOA method at  $t_{n-1}$ , ( $\mathbf{P}_i = \mathbf{P}_{n-1, AOA}$ ). As shown in Fig. 5, with all three initial positions, the PuOB method outperforms other methods considered in this work. According to results, if the distance between the initial position and intended position is more than

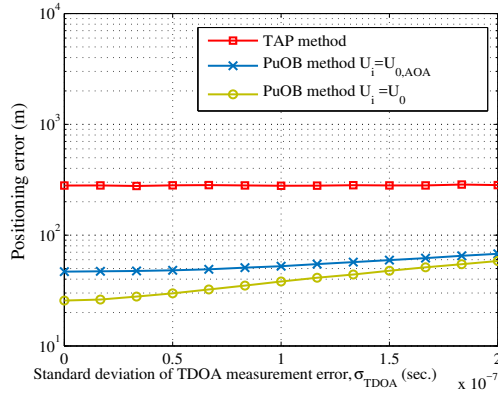


Fig. 6: The effect of  $\sigma_{TDOA}$  variation on the performance of PuOB, AOA and TAP methods.

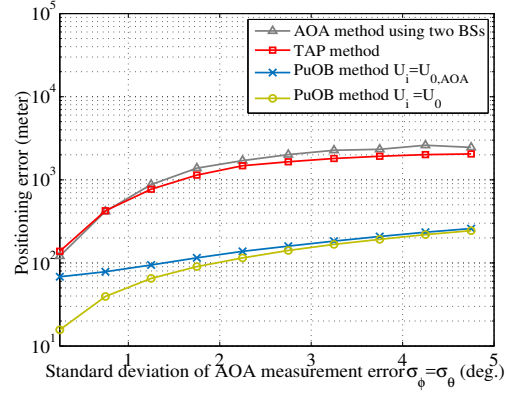


Fig. 7: The effect of  $[\sigma_\phi \ \sigma_\theta]^T$  variations on the performance of PuOB, AOA and TAP methods ( $\sigma_\phi = \sigma_\theta$ ).

5000 m, the dependency of PuOB estimation on initial position will be reduced. Furthermore, as mentioned in first scenario, AOA method performs slightly better than TAP method in y-axis positioning because of clear AOA measurements of transmitter at BSs located in x-axis. However, Figs. 4 (b) and (c) show that TAP method performs better than AOA method generally.

In the third scenario, we assume that  $\sigma_{TDOA}$  is variable while the locations and  $[\sigma_\phi \ \sigma_\theta]^T$  are fixed.

$\sigma_{TDOA}$  is varied from 0 to 200 ns to observe the performance of the PuOB method and TAP methods. The location of the transmitter is fixed in  $\mathbf{P}_n = [2000 \ 1000 \ 1000]^T$  and  $\mathbf{P}_0$  is set to  $[1000 \ 1000 \ 1000]^T$ .  $[\sigma_\phi \ \sigma_\theta]^T$  are fixed as  $[0.5 \ 0.5]^T$  degrees. Two cases of initializations are considered like the first two initializations in the second scenario. First,  $\mathbf{P}_i = \mathbf{P}_0$  and second,  $\mathbf{P}_i = \mathbf{P}_{0, AOA}$ . Fig. 6 depicts that the PuOB method with both of initialization procedures outperforms TAP method. Even if PuOB method is initialized by erroneous position obtained by AOA estimation, its performance approaches to the estimations initialized by the exact position of the transmitter. Furthermore, TAP method results in a relatively constant error performance in low variations of  $\sigma_{TDOA}$  due to its low dependency on TDOA measurements.

The last scenario is reserved to study the effect of  $[\sigma_\phi \ \sigma_\theta]^T$  variations on the performance of localization methods. We assume  $[\sigma_\phi \ \sigma_\theta]^T$  vary from 0.25 degrees to 5 degrees.  $\sigma_{TDOA}$  is set to 30 ns. The position of the transmitter and initial positions ( $\mathbf{P}_i$ ) are the same as the third scenario. As shown in Fig. 7, PuOB method outperforms AOA and TAP methods. It is worth mentioning that both of the initialized PuOB estimation cases approach to each other in high  $[\sigma_\phi \ \sigma_\theta]^T$  variations. Furthermore, the TAP method outperforms AOA method after  $[\sigma_\phi \ \sigma_\theta]^T = [1 \ 1]^T$  degrees.

#### IV. CONCLUSIONS

In this paper, a simple and linear positioning method, namely PuOB is introduced, which requires only one base station to track the location of mobile transmitters. Simulation results indicate that PuOB with one base station has superior

positioning performance than the angle-of-arrival and hybrid time and angel of arrival methods with two base stations. Additionally, PuOB is robust against errors in angle and time measurements.

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