Compensation for the Mutual Coupling in Transmitting Antenna Arrays

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***Abstract*—A numerical study is presented for the compensation of mutual coupling in antenna arrays. A Matlab code based on method of moments is used to find the compensated far field radiation patterns for non-identical and/or staggered wire antenna arrays. The isolated patterns for some antenna arrays when applicable are found using the principle of pattern multiplication which are used to predict the compensated patterns.**

Key words— Mutual Coupling, Method of Moments, Antenna Arrays

I. INTRODUCTION

It is well known that the presence of mutual coupling between the elements of a transmitting antenna array limits the simple use of array factor method. To obtain a real desired pattern one usually needs to modify the excitation voltages applied to individual antennas using the array factor method. This is because of the fact that array factor method assumes no mutual coupling between the individual antenna elements. In [1], [2] and [6] some techniques are suggested for compensation of these modified (compensated) excitation voltages. Both [1] and [2] use commercial software to calculate compensated voltages for linear arrays of identical dipole antenna elements.

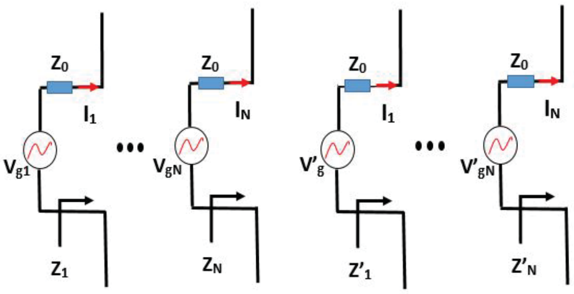
In this work an in-house full wave electromagnetic solver is developed. A Matlab code based on the Method of Moments (MoM) is written to study the cases of identical wire antenna arrays presented in [1] and [2]. Furthermore, arrays of non-identical and/or staggered dipole antennas were considered i.e., the antenna elements (lengths, radius, positon) can be different. Results for such arrays could not be found in the literature and a simple array factor theory does not apply to such cases. Our results for the mutual impedance of non-identical staggered linear elements were compared with those of [3] which considered dipole antennas of zero radius. The code first computes the scattering matrix of the array and then the values of the compensated excitation voltages.

II. COMPENSATION METHOD

The pattern of an array of uniform linear antennas (ULA) can be changed by fixing the input current I_{in}. The input currents without mutual coupling for N center-fed thin wire antennas can be found as

$$I_n = \frac{V_{gn}}{Z_0 + Z_n} \quad \text{for n=1,2..N.} \tag{1}$$

Here V_{gn} is the generator voltage feeding the n_{th} antenna, $Z_0 = 50\ \Omega$ is the internal resistance of the source, and Z_n is the input impedance of the n_{th} antenna when it is isolated, that is, when the other N-1 antennas are removed. Figure 1a shows the equivalent circuit. When mutual coupling exists the input impedance of each individual antenna changes to $Z_n^{'}$, thus changing the input currents. In order to bring the currents back to the desired values I_n, the generator voltage must be readjusted to $V_{gn}^{'}$. The input impedance have changed from Z_n to $Z_n^{'}$ whereas the input currents remain the same as shown in Fig. 1b. Thus the source voltages are changed from V_{gn} to $V_{gn}^{'}$. The desired current can be written as



(a) (b)

Fig. 1. Equivalent circuit of N antennas (a) with no mutual coupling (b) with mutual coupling

$$I_n = \frac{V_n^+ - V_n^-}{Z_0} \tag{2}$$

where V_n^+ is the forward (incident) voltage entering the n_{th} antenna of the N-port network defined by

$$V_n^+ = \frac{V_{gn}^{'}}{2} \tag{3}$$

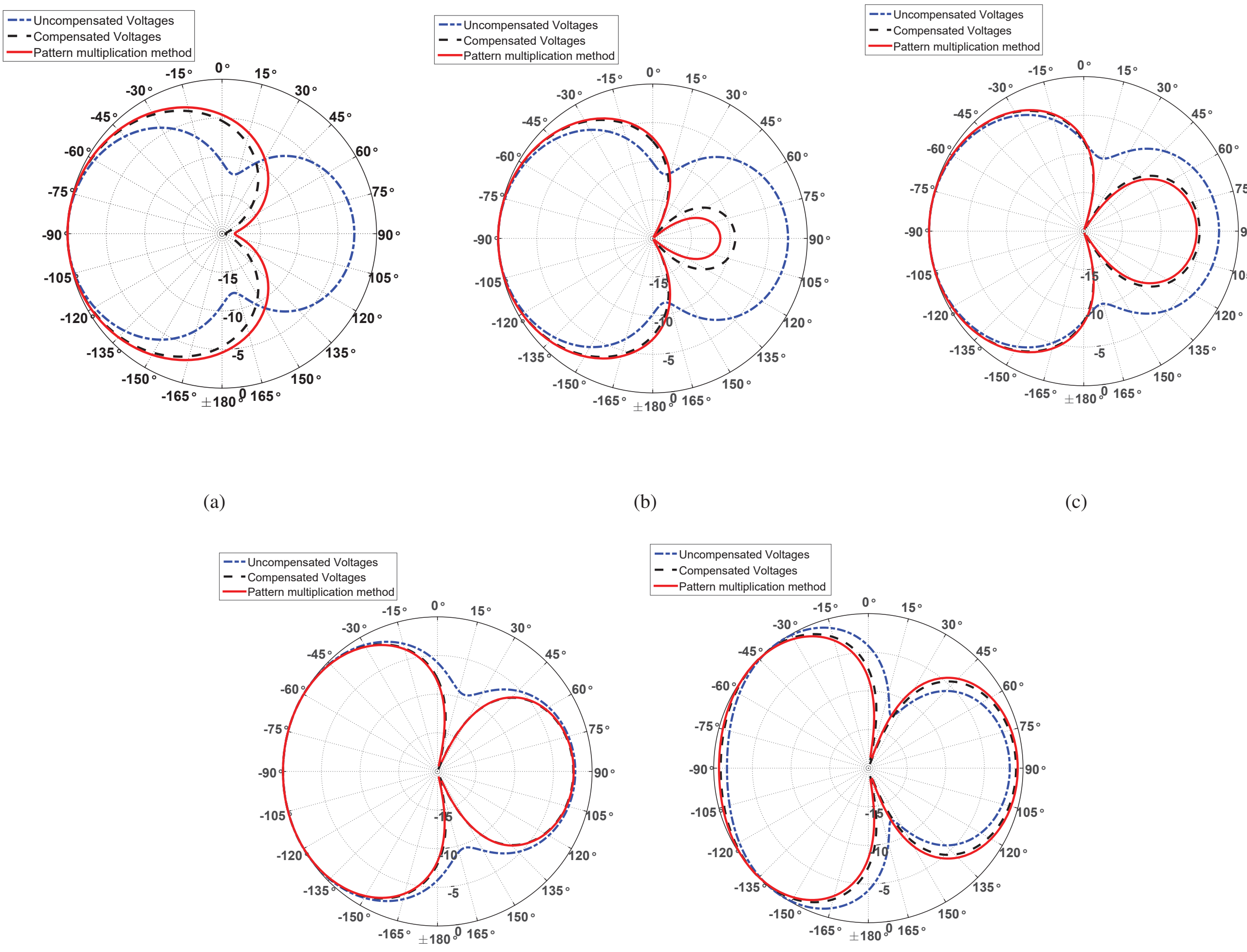
and V_n^- is the reflected voltage from the n_{th} antenna as

$$V_n^- = S_{n1}V_1{}^+ + S_{n2}V_2{}^+ + \cdots + S_{nN}V_N{}^+ \tag{4}$$

Where, S_{ij} is the element of the scattering matrix for the system. Substituting the values of (3) and (4) in (2) we get

$$[I] = \frac{1}{2Z_0}\{U - S\}^{-1}[V_g^{'}] \tag{5}$$

Here [I] is the Nx1 column vector of the desired input currents. U is NxN unit matrix and $[Vg^{'}]$ is the Nx1 column vector of the desired i.e., compensated source voltages feeding



(a) (b) (c)

(d) (e)

Fig. 2. Radiation pattern of two dipole array with different element separations. (a) $d = 0.1\lambda$ (b) $d = 0.2\lambda$ (c) $d = 0.3\lambda$ (d) $d = 0.4\lambda$ (e) $d = 0.5\lambda$

Table I
COMPARISON OF THE NORMALISED COMPENSATION VOLTAGES OF A TWO ELEMENT DIPOLE ARRAY FOR DIFFERENT ANTENNA SEPARATIONS

	$d = 0.1\lambda$	$d = 0.2\lambda$	$d = 0.3\lambda$	$d = 0.4\lambda$	$d = 0.5\lambda$
reported in [1]	$0.99\angle 70.49°$	$0.613\angle 95.33°$	$0.572\angle 121.92°$	$0.659\angle 140.02°$	$0.754\angle 141.16°$
reported in (1) [2]	$0.773\angle 31.16°$	$0.466\angle 105.21°$	$0.590\angle 129.85°$	$0.681\angle 140.07°$	$0.774\angle 146.80°$
reported in (2) [2]	$0.912\angle 69.57°$	$0.578\angle 97.33°$	$0.562\angle 125.78°$	$0.659\angle 141.13°$	$0.781\angle 147.86°$
this paper	$0.5901\angle 70.36°$	$0.449\angle 114.37°$	$0.5679\angle 139.6°$	$0.713\angle 148.44°$	$0.850\angle 151.40°$

the antennas. The desired adjusted voltage source values (in the presence of mutual coupling) are given by

$$[V_g^{'}] = (2Z_0)\{U - S\}^{-1}[I] \tag{6}$$

III. NUMERICAL METHOD AND RESULTS

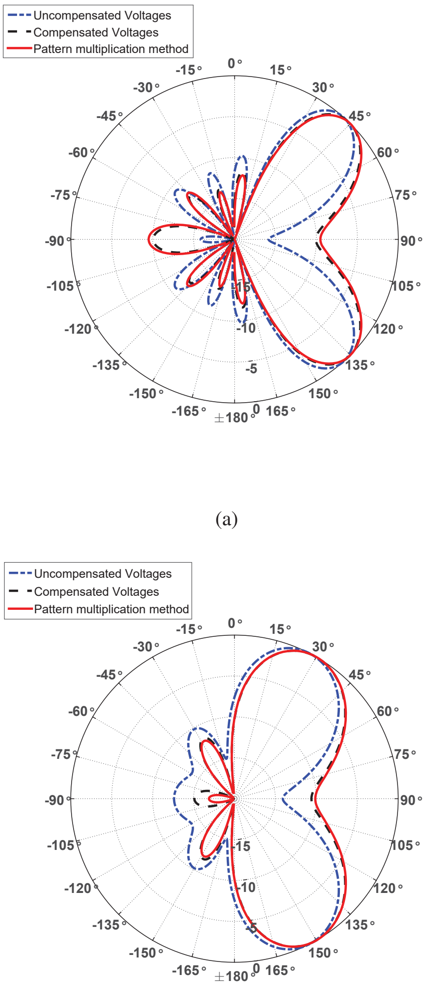
In the analysis of MoM, piecewise sinusoidal (PWS) functions are used as expansion functions. Testing is done using the Galerkin method. The moment matrix elements are computed using closed form of the integral in [4] using the Si and Ci functions. Using this matrix and its inverse one can compute Z, Y and S parameters for the array. A magnetic frill current is used for the excitation of the individual antenna element [5].

A. Two-element dipole Array

First, a two-element dipole antenna array similar to that of [1] with length $\lambda/2$ and radius $\lambda/200$ is used. The number of expansion functions per dipole is 7. The magnetic frill source is at the center of each wire element. The antenna element spacing is varied from 0.1λ to 0.5λ. The source internal impedance Z_0 is $50\ \Omega$ and the original excitation voltage sources are $V_{g1} = 1\ V$ and $V_{g2} = 1\angle 135° V$, respectively. The normalised compensated voltages are tabulated in Table I along with the results of [1] and [2]. Our results are in close agreement with [1] and [2]. The resultant far field patterns in Figs. 2(a-e) show the far field patterns due to the uncompensated voltages, compensated voltages and pattern multiplication method. It can be observed that the array patterns due to the compensated voltages is almost the same as that of the isolated pattern results computed using pattern multiplication method.

B. Five-element dipole array

Five element dipole ULA in two different configurations is studied similar to that in [1] and [2]. Same parameters for length, radius and internal source impedances are used as in the case of two-element dipole arrray except for excitation voltages and element spacing. In the first case, we use $d = 0.5\lambda$ as element spacing and main-beam direction is excited at $\phi = 45°$. The original excitation voltages are identical to Table 3 of [1]. For the second case $d = 0.3\lambda$ and $\phi = 60°$. The resultant far field patterns are shown in Figs. 3a and 3b. The normalized compensated voltages for both cases are tabulated in Table II and are in agreement with [1] and [2].



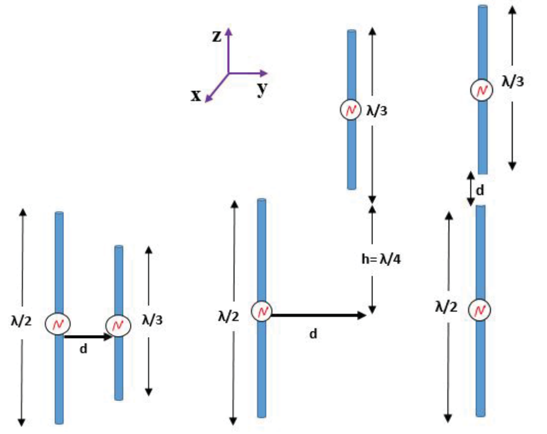
(a)

(b)

Fig. 3. Radiation pattern for five-element dipole array with different separations and main-beam directions (a) $d = 0.5\lambda$, $\phi = 45°$ (b) $d = 0.3\lambda$, $\phi = 60°$

Table II
NORMALISED COMPENSATION VOLTAGES OF FIVE-ELEMENT DIPOLE ARRAY

	$d = 0.5\lambda, \phi = 45°$	$d = 0.3\lambda, \phi = 60°$
V'_{s2}/V'_{s1}	$1.394\angle -132.24°$	$1.554\angle -66.07°$
V'_{s3}/V'_{s1}	$1.616\angle 92.60°$	$1.680\angle -121.28°$
V'_{s4}/V'_{s1}	$1.712\angle -44.39°$	$1.691\angle 166.0°$
V'_{s5}/V'_{s1}	$1.615\angle 174.00°$	$1.637\angle 122.70°$



(a) (b) (c)

Fig. 4. Two parallel antennas in (a) non-staggered (b) staggered h=λ/4 (c) collinear arrangement.

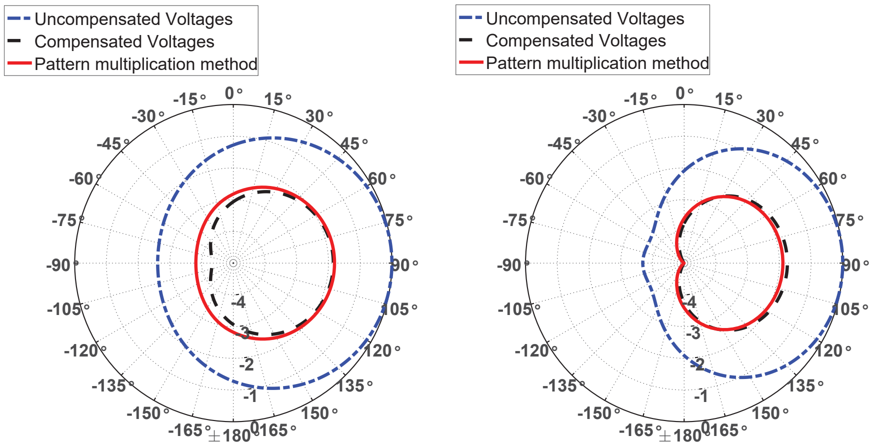
C. Two-element non-identical non-staggered dipoles

A two-element non-identical dipole antenna array with lengths $L_1 = \lambda/2$ and $L_2 = \lambda/3$ with similar radius $\lambda/1000$ is used. The number of expansion functions for the wires are N_1=9 and N_2=7.

Table III
COMPENSATION VOLTAGES V'_{s1} AND V'_{s2} OF TWO-ELEMENT NON-IDENTICAL NON-STAGGERED DIPOLE ARRAY FOR DIFFERENT ANTENNA SEPARATIONS

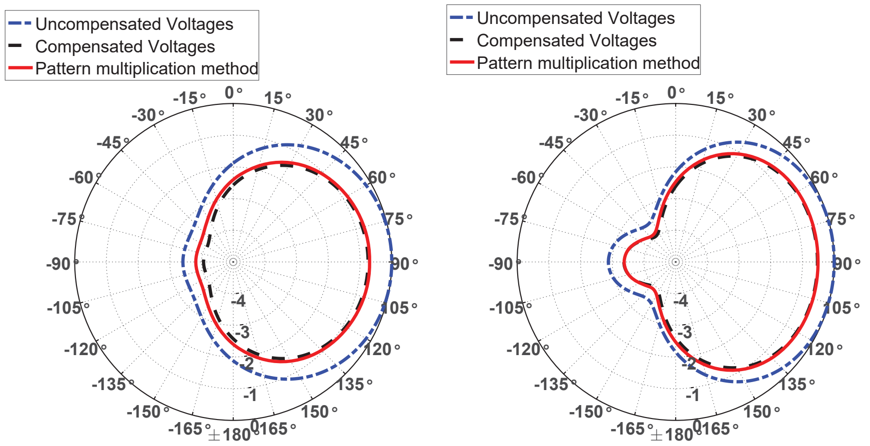
Antenna separation $d(\lambda)$	$V'_{s1}, (V)$	$V'_{s2}, (V)$
0.1	$0.8199\angle -6.46°$	$0.7517\angle 128.0°$
0.2	$0.8307\angle -0.65°$	$0.7723\angle 135.2°$
0.3	$0.8752\angle 3.56°$	$0.8314\angle 140.7°$
0.4	$0.9360\angle 5.29°$	$0.9117\angle 142.9°$
0.5	$0.9921\angle 4.92°$	$0.9877\angle 142.3°$

The antennas are placed parallel and non-staggered as in Fig. 4a. The magnetic frill source was at the center of



(a) (b)

Fig. 5. Radiation pattern of two-element non-identical non-staggered dipoles with different element separations. (a) $d = 0.1\lambda$ (b) $d = 0.2\lambda$



(a) (b)

Fig. 6. Radiation pattern of two-element non-identical staggered dipoles with different element separations. (a) $d = 0.2\lambda$ (b) $d = 0.3\lambda$

each wire element. The antenna element spacing is varied from 0.1λ to 0.5λ. The excitation voltage sources are set to $V_{g1} = 1\ V$ and $V_{g2} = 1\angle 135° V$, respectively. The results for the compensated voltages are tabulated in Table III. Figures 5a and 5b show the far field patterns for $d = 0.1\lambda$ and $d = 0.2\lambda$ due to the uncompensated voltages, compensated voltages and pattern multiplication method. Figure 5b shows that the compensation becomes negligible as d increases. Note that in this case of non-identical elements, *pattern multiplication* method actually consists of adding the patterns of the isolated individual antennas excited by original sources.

D. Two-element non-identical staggered dipoles

A two-element non-identical staggered dipole antenna array with lengths $L_1 = \lambda/2$ and $L_2 = \lambda/3$ with similar radius $\lambda/1000$ is used. The number of expansion functions for the wires are N_1=9 and N_2=7. The antennas are staggered by $h = \lambda/4$ as in Fig. 4b. The magnetic frill source was at the center of each wire element. The antenna element spacing is varied from 0.1λ to 0.5λ. The excitation voltage sources are $V_{g1} = 1\ V$ and $V_{g2} = 1\angle 135° V$, respectively. The results for the compensated voltages are tabulated in Table IV. Figures 6a and 6b show the far field patterns for $d = 0.2\lambda$ and $d = 0.3\lambda$ due to the uncompensated voltages, compensated voltages, and pattern multiplication method.

Table IV
COMPENSATION VOLTAGES $V_{s1}^{'}$ AND $V_{s2}^{'}$ OF TWO-ELEMENT NON-IDENTICAL STAGGERED DIPOLE ARRAY FOR DIFFERENT ANTENNA SEPARATIONS

Antenna separation $d(\lambda)$	$V_{s1}^{'}, (V)$	$V_{s2}^{'}, (V)$
0.1	$0.9278\angle -4.83°$	$0.8893\angle 129.1°$
0.2	$0.9230\angle -0.81°$	$0.8902\angle 134.5°$
0.3	$0.9453\angle 1.71°$	$0.9221\angle 137.9°$
0.4	$0.9782\angle 2.62°$	$0.9670\angle 139.1°$
0.5	$1.0081\angle 2.27°$	$1.0082\angle 138.5°$

It is evident that in case of non-staggered and staggered arrangement the mutual coupling and compensation voltages are different. Our results for the mutual impedance of many staggered antennas have been verified with those in [3]. Furthermore, the mutual impedance between two identical staggered antennas, with length $L = 0.4781\lambda$, radius $a = 0.001\lambda$ and a fixed antenna spacing of $d = 0.25\lambda$ as mentioned in Fig.3-28 [4], was computed. We propose the correct results as shown in Fig.7 and disagree with those of [4].

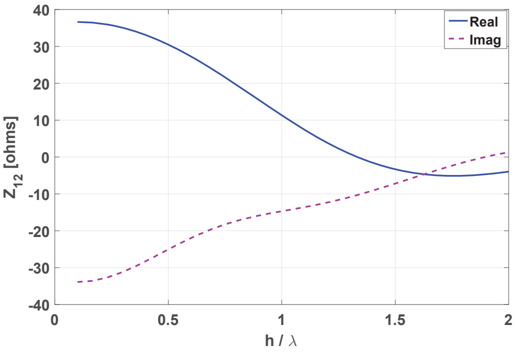


Fig. 7. The mutual impedance between two identical staggered dipoles ($L = 0.4781\lambda$, $a = 0.001\lambda$, $d = 0.25\lambda$) as a function of staggered spacing h relative to wavelength.

E. Two-element non-identical collinear dipoles

Next, the two-element antenna array considered in C and D are placed in a collinear arrangement as shown in Fig. 4c. The antenna element spacing d is varied from 0.1λ to 0.5λ. The excitation voltage sources are $V_{g1} = 1\ V$ and $V_{g2} = 1\angle 135° V$, respectively. The results for the compensated voltages are tabulated in Table V. As the antennas are collinear the mutual coupling is weak which is evident by the results.

Table V
COMPENSATION VOLTAGES $V_{s1}^{'}$ AND $V_{s2}^{'}$ OF TWO-ELEMENT NON-IDENTICAL COLLINEAR DIPOLE ARRAY FOR DIFFERENT ANTENNA SEPARATIONS

Antenna separation $d(\lambda)$	$V_{s1}^{'}, (V)$	$V_{s2}^{'}, (V)$
0.1	$0.9339\angle -2.2°$	$0.9018\angle 132.7°$
0.2	$0.9502\angle 0.19°$	$0.9274\angle 135.9°$
0.3	$0.9707\angle 0.97°$	$0.9558\angle 136.8°$
0.4	$0.9869\angle 0.93°$	$0.9777\angle 136.7°$
0.5	$0.9965\angle 0.56°$	$0.9905\angle 136.2°$

IV. CONCLUSION

The mutual compensation computed by [1] and [2] have been verified using a Matlab code that uses Method of Moments with PWS sinusoid and Galerkin method with magnetic frill as the source of excitation. Furthermore, mutual compensation for non-identical and/or staggered antennas have been computed. The results show that the code is effective for both strong and weak mutual coupling compensation.

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