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# Assessment of hydrogen production methods for global energy transition using AI enhanced quantum recommender fuzzy modelling

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ABSTRACT

The main performance indicators of hydrogen energy production should be improved. However, improving these factors also increase the operational costs of the companies. Because of this issue, there is a need for a priority analysis so that it can be possible to focus on more important factors. Accordingly, the purpose of this study is to evaluate hydrogen production methods for global energy transition. In this process, a four-stage model has been proposed by getting evaluations from three different experts. Firstly, artificial intelligence-based decision-making can be implemented for expert prioritization. In the second stage, recommender system is conducted with collaborative filtering to complete the missing evaluations. Thirdly, selected criteria are weighted by using M-SWARA with QPFRS. Finally, method alternatives for hydrogen production are ranked via quantum picture fuzzy rough sets adopted VIKOR. The biggest contribution for doing this study is that artificial intelligence technique is integrated into the model and experts' importance coefficients are can be computed. Additionally, by using the collaborative filtering technique, empty evaluations can be filled scientifically. This contributes to the quality of the analysis process in many ways. Thanks to this technique, experts are given the opportunity not to answer questions they are not very sure about. The findings indicate that renewable energy expansion, energy efficiency and sustainable development are the most important criteria for global energy transition in hydrogen production. On the other side, the ranking results give information that thermal processes including steam methane reforming and biomass gasification is the most appropriate method alternatives for hydrogen production. Based on these analysis results, it is strongly recommended that research and development activities should be improved to increase the efficiency and effectiveness of the renewable energy projects. With the help of this issue, it can be much easier to increase the performance of hydrogen production process.

## 1. Introduction

Hydrogen energy production is very important for future energy conversion and sustainability. The most important advantage of hydrogen energy production is that it is a clean energy source. It is known as an environmentally friendly energy source because it produces water vapor instead of carbon emissions during the energy production process. This plays an important role in the fight against climate change. Another advantage of hydrogen energy is that it can be used to store and transport energy [1]. Hydrogen energy production is of great importance for global energy transformation [2]. For these reasons, hydrogen energy production and use is becoming a critical component of the global energy transition. To achieve global energy conversion in hydrogen production, many different variables need to be taken into consideration. Decarbonization is one of the most important issues in this process. Therefore, the production of decarbonized hydrogen plays an important role in combating climate change [3]. Similarly, in hydrogen production, renewable energy expansion is extremely important for the success of the global energy transition. Moreover, in terms of energy storage, hydrogen production can contribute to global energy transformation [4]. The energy storage process plays a vital role in the development of renewable energy projects. In this process, excess renewable energy produced by hydrogen production can be stored.

These factors need to be improved for hydrogen energy production to

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contribute to the global energy transformation. However, improving these factors also leads to increased costs. Therefore, it is financially better to focus on the more important factors rather than improving all factors. In this way, it is possible to achieve efficiency while applying the right policies. There are also some different method alternatives for hydrogen production [5]. Thermal processes including steam methane reforming and biomass gasification can be taken into consideration in this process. Similarly, electrolytic processes splitting water into hydrogen and oxygens with water electrolysis and photolytic processes using photobiological facilities and photoelectrochemical cells play a key role in this regard. Additionally, methane pyrolysis and thermochemical processes using solar energy to produce hydrogen from water or hydrocarbons is another method for this situation [6]. In this process, the main problem is that all improvements create new operational costs for the companies. Due to this situation, it is not financially feasible for these companies to make improvements for all issues. Hence, more critical factors should be identified so that the solutions can be implemented in a more efficient manner. However, most of the studies in the literature examine the important indicators of hydrogen energy production process. Nevertheless, there are limited studies in the literature that focused on this issue. This situation can be accepted as the main gap in the literature with respect to the subject of hydrogen production.

Accordingly, in this study, it is aimed to evaluate hydrogen production methods for global energy transition. Thus, the main research question of this study is which factors should be prioritized to increase the effectiveness of hydrogen energy production processes. For this purpose, a four-stage model has been proposed. Firstly, AI-based decision-making can be implemented for expert prioritization. In the second stage, recommender system is conducted with collaborative filtering to complete the missing evaluations. Thirdly, selected criteria are weighted by using M-SWARA with QPFRS. Finally, method alternatives for hydrogen production are ranked via QPFRS adopted VIKOR. The biggest motivation for doing this study is that it is necessary to establish a new and comprehensive fuzzy decision-making model to determine the most effective hydrogen energy production method. Models that currently exist in the literature can be criticized in many aspects. One of the most important issues in this process is that the importance of experts is considered equal in most of these models. On the other hand, experts should have different coefficients due to their different demographic characteristics, such as education level and working experience. To eliminate this criticism, in this study, AI technique is integrated into the model and experts' importance coefficients are can be computed. This situation has a positive contribution to the effectiveness of the analysis results.

The main contributions of this article are denoted below. (i) By using the collaborative filtering technique, empty evaluations can be filled scientifically. This contributes to the quality of the analysis process in many ways. Thanks to this technique, experts are given the opportunity not to answer questions they are not very sure about. Otherwise, when collaborative filtering is not used, experts have to evaluate even questions they are not very sure about. This situation leads to a decrease in the accuracy of analysis processes. (ii) Using the M-SWARA technique in determining the importance weights of variables also provides some advantages. This technique is achieved by making some improvements to the classical SWARA method. Thanks to these improvements, both criterion weights are calculated and causal relationships between criteria are taken into account. Factors affecting the effectiveness of energy conversion in hydrogen production may have a causal effect on each other. Therefore, to achieve more accurate analysis results, the M-SWARA technique is one of the most optimal methods that can be considered in this process. (iii) The integration of AI methodology to the proposed model contributes the methodological originality. This integration provides opportunity to compute the weights of the decision makers. In other words, the decision makers who have more working experience and better education level can have greater weights. This situation makes a powerful contribution to the effectiveness and appropriateness of the analysis results.

The second section gives information about the literature review. The third section includes the details of the proposed model. Analysis results are indicated in the following section. The final sections consist of discussion and conclusion.

## 2. Literature review

Storage and transportation difficulties are among the important criteria in determining hydrogen technology. For hydrogen to become a low- or zero-carbon energy carrier, storage and transportation deficiencies must be eliminated [7]. Safe storage of hydrogen accelerates the transition to a low-carbon energy system [8,9]. As a matter of fact, Ma et al. [10] emphasized in their study the importance of government support in green hydrogen storage. Asif et al. [11] identified that hydrogen should be converted to ammonia as a way to store hydrogen effectively and safely. Additionally, Qureshi et al. [12] emphasized that the transportation and storage of hydrogen is a critical problem. Furthermore [13], highlighted that the existing infrastructure system and policy frameworks should be improved to transport hydrogen safely and efficiently. In addition to this issue, safety concern is another important criterion this process [14]. Zainal et al. (2023) emphasized that a robust hydrogen infrastructure must be developed to prevent vulnerabilities encountered in hydrogen production. Guo et al. [15] pointed out that due to the properties of hydrogen, it is difficult to detect even if there is a leak.

Adequate legal framework is one of the important criteria in determining effective hydrogen technology. Legal frameworks generally include international agreements and decarbonization regulations implemented by countries. For this purpose [16], and Seyyedattar et al. [17] defined that sustainable development goals can prevent social and ecological injustice in the planning of hydrogen projects. However, Bade et al. [18] emphasized that despite significant investments in hydrogen technology, there are many deficiencies such as economic efficiency, social acceptance and legal regulation. In addition, it is identified that there are more legal regulations regarding hydrogen technology in Europe and Asia compared to the USA. Cost effectiveness is also among the effective methods in determining effective hydrogen technology. Investment amounts vary for different hydrogen production technologies. Cost analysis of different hydrogen production technologies is based on economies of scale [19]. Zhiznin et al. [20] highlighted that hydrogen produced using the electrolyzer method significantly increases the production cost. Harichandan et al. [21] stated that financing in green hydrogen production projects requires the combination of the public and private sectors.

Green hydrogen production technologies are seen as an effective way to reduce carbon emissions. Therefore, the number of investments in this field is gradually increasing. It is important that the technological infrastructure is at a sufficient level in the development of green hydrogen technologies. If countries have sufficient technological infrastructure, they will offer more efficient hydrogen technologies [22]. Having advanced technology will help to quickly eliminate any problems that may occur in hydrogen production. Ampah et al. [23] evaluated the latest studies in hydrogen production in their study. They concluded that increasing the budget allocated for R&D encourages innovations in hydrogen production. Zhang et al. [24] stated that to develop hydrogen technologies, various production techniques, transportation and storage technologies, and the hydrogen potential in the sector should be reviewed. Pleshivtseva et al. [25] and Su et al. [26] pointed out that green hydrogen production methods are increasing. They emphasized that the amount of hydrogen obtained with this production method approaches the amount produced from fossil fuels. They reached a conclusion that this situation could be an important step for decarbonization.

Experiencing energy losses constitutes one of the possible criteria in determining effective hydrogen technology. These losses directly affects the effectiveness, sustainability and efficiency of hydrogen production [27]. With all these aspects, preventing energy losses will contribute to making hydrogen production more effective among renewable energy technologies [28]. For this purpose, Tang et al. [29] determined that as hydrogen production methods differ, energy losses also differ. They emphasized that thermochemical and electrolysis methods need to be developed to use hydrogen in climate change. In addition, Cormos [30] concluded that energy losses are reduced, and costs are reduced in hydrogen production using membranes. In addition, Martins et al. [31] identified that the hydrogen obtained through biomass gasification is based on the maximum hydrogen yield. They also state in their studies that there is increasing interest in gasification as a cleaner and sustainable method to produce green hydrogen.

It is possible to obtain some important points as a result of the literature review. The demand for clean energy is increasing, especially due to the increasing carbon emission problem. In this context, it seems that the popularity of hydrogen energy is increasing. In this context, the importance of producing hydrogen energy is increasing significantly. On the other hand, the effectiveness of these investments must be ensured to ensure continuity in hydrogen energy production. However, there are many variables that can have an impact on the performance of these projects. However, improving these variables also leads to increased costs. In other words, it is not financially possible to improve many variables. Therefore, priority should be given to variables that are more important. Nevertheless, there are a limited number of studies in the literature that conducted priority analysis for these criteria. This situation can be defined as the most important gap in the hydrogen energy production literature. To fill this gap in the literature, this study aims to create a new fuzzy decision-making model and perform priority analysis for these variables.

## 3. Proposed model

The subject of the article is the ranking of alternative methods for hydrogen production. It is necessary to identify and weight effective criteria for the global energy transition in hydrogen production. Multi-criteria decision-making techniques are preferred in ranking alternatives and weighting criteria. While the M-SWARA method is preferred for weighting the criteria in hydrogen production, the alternatives are ranked by the VIKOR method. In addition, since the two methods are based on expert opinions, linguistic ambiguity is included in the analysis. For this purpose, fuzzy set theory, a mathematical theory that deals with uncertainty, is used. Another issue is the prioritization of experts and completion of missing evaluations. For these situations, artificial intelligence models are used. While experts are prioritized with the K-means clustering algorithm, missing data are estimated with the collaborative filtering method.

The stages and steps of the proposed model for determining alternative methods for hydrogen production are summarized in Fig. 1.

Each stage in proposed model is detailed under subtitles.

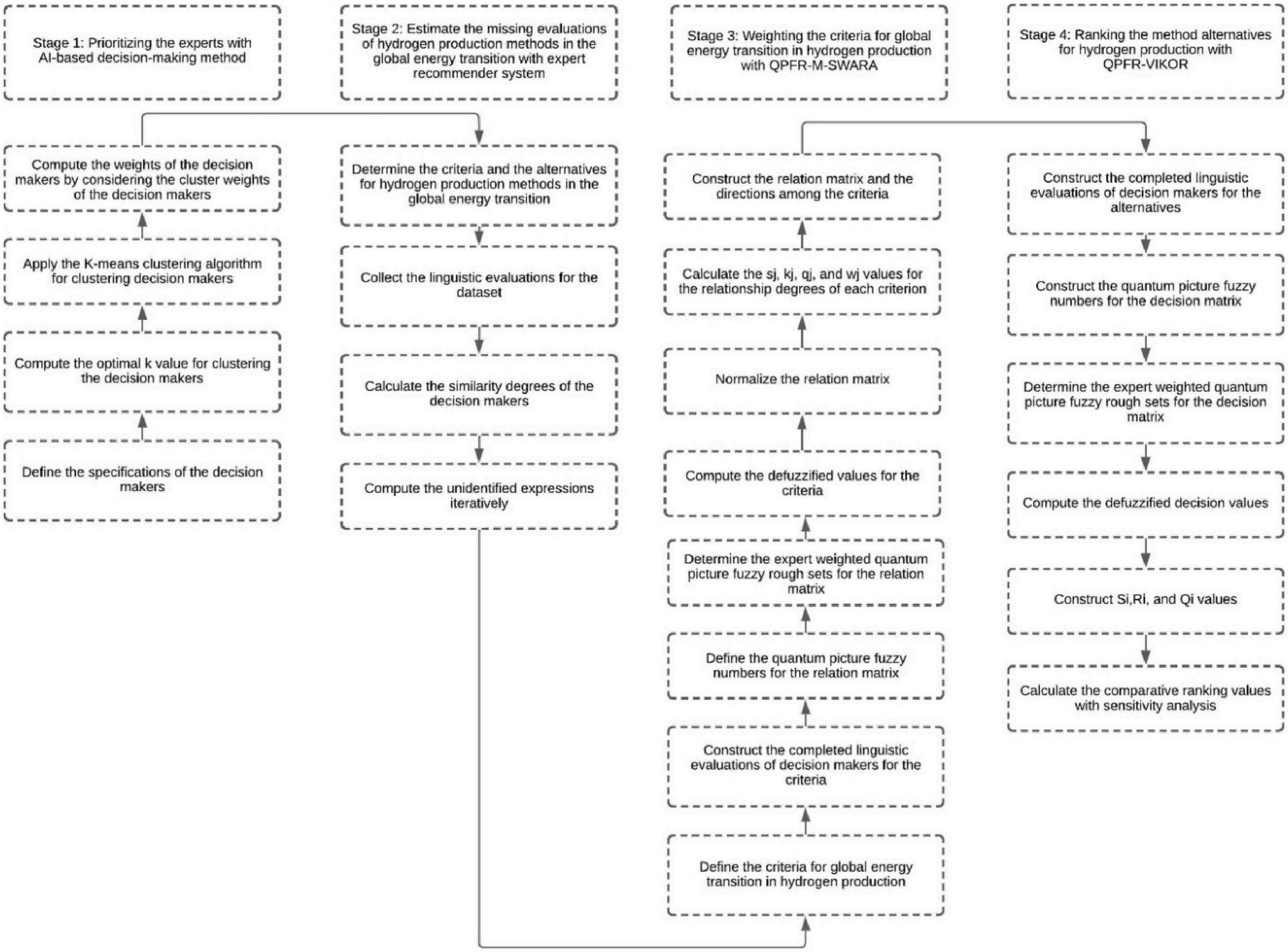

**Fig. 1.** The flowchart of hybrid model.

### *3.1. AI-based decision-making for expert prioritization*

One of the most common objections in the literature is the assumption that expert assessments in multi-criteria decision-making approaches are equally important. The primary source of this is that each expert has a varied level of knowledge due to differences in their backgrounds, experiences, and other factors. The essay suggests using artificial intelligence to prioritize experts in response to these objections. The k-means clustering algorithm serves as the foundation for the suggested model. The following provides information on each of the suggested model's four steps.

*Step 1* involves defining of specifications of the decision makers. A data set (X) containing information such as education, salary, age of decision makers is created. *Step 2* is about calculating of the Within-Cluster Sum of Squares (WCSS) with various numbers of cluster (k) [32]. The WCSS values are calculated by Equation (1).

$$WCSS=\sum_{j=1}^{k}\sum_{x_i\in C_j} d\left(x_i,c_j\right)^2 \tag{1}$$

A plot is drawing showing WCSS values computed for the elbow point. The elbow point is the break point in the plot, and the optimal number of clusters is equal to the horizontal axis value of this point. *Step 3* is about applying the k-means clustering algorithm for clustering experts. Using Equations (2) and (3), the process is repeated until no data point's cluster membership $(x_{jl})$ changes or the maximum number of iterations is reached.

$$d\left(x_i,x_j\right)=\sqrt{\sum_{l=1}^{n}\left(x_{il}-x_{jl}\right)^2} \tag{2}$$

$$c_j=\frac{1}{|C_j|}\sum_{x_i\in C_j} x_i \tag{3}$$

$C_j$ represents the set of data points in jth-cluster, while $|C_j|$ equals data points' number in jth-cluster. *Step 4* is about computing of DMs' weights by considering the DMs' cluster weights. The mean standard deviations $(s_j)$ of each cluster is calculated by Equations (4)–(6).

$$s_j=\frac{1}{n}\sum_{l=1}^{n}\sigma_{jl} \tag{4}$$

$$\sigma_{jl}=\sqrt{\frac{1}{|C_j|}\sum_{x_i\in C_j}\left(x_{il}-x_{jl}\right)^2} \tag{5}$$

$$x_{jl}=\frac{1}{|C_j|}\sum_{x_i\in C_j} x_{il} \tag{6}$$

$\sigma_{jl}$ means the standard deviation of $l$-feature in jth-cluster. $x_{jl}$ equals the average of $l$-feature in jth-cluster. Afterwards, the cluster weights $(w_j)$ are computed with Equation (7).

$$w_j=|C_j|\times s_j \tag{7}$$

where $|C_j|$ means the size of jth-cluster. Finally, the weights of decision makers $(w_{tj})$ are determined using Equation (8).

$$w_{tj}=\frac{1}{|C_j|}\frac{w_j}{\sum\limits_{w_j\in C_j} w_j} \tag{8}$$

Where, $t$ represents the number of DMs and $w_{tj}$ is the DM's weight $t$ in jth-cluster.

### *3.2. Recommender system with collaborative filtering*

There could be values missing from the compilation of opinions. Experts may occasionally refrain from speaking up and offer no view. Requiring specialists to perform assessments or gathering data again could have an impact on how reliable the analysis's conclusions are. To fill in the gaps in the data, the Collaborative Filtering method is advised. Below is an outline of the four-step procedure [33].

*Step 5* involves determining the criteria and the alternatives for hydrogen production methods in the global energy transition. The criteria and alternatives sets are collected in result of literature review. In *Step 6* linguistic opinions for the dataset is collected. Linguistic opinions for analysis are collected from expert team. *Step 7* covers calculating the similarity degrees of the decision makers with Equation (9). The reason is that it is necessary to apply the collaborative filtering technique.

$$sim(u,v)=\frac{\sum_{i\in I}\left(r_{u,i}-\overline{r_u}\right)\left(r_{v,i}-\overline{r_v}\right)}{\sqrt{\sum_{i\in I}\left(r_{u,i}-\overline{r_u}\right)^2}\sqrt{\sum_{i\in I}\left(r_{v,i}-\overline{r_v}\right)^2}} \tag{9}$$

Where, $r_{u,i}/\ r_{v,i}$ represent the rating degrees of decision makers and $\overline{r_u}$ and $\overline{r_v}$ are the averaged values. *Step 8* is about computing unidentified expressions iteratively by Equation (10).

$$p_{u,i}=\frac{\sum_{j\in S} sim(u,v) r_{u,j}}{\sum_{j\in S}|sim(u,v)|} \tag{10}$$

### *3.3. Modelling uncertainty with QPFRS with golden cuts*

One of concepts that contain ambiguity are known as linguistic evaluation. Fuzzy set theory is a branch of mathematics that is advised when working with words that include uncertainty. One of the set theories created to quantify uncertainty and incorporate it into analysis is fuzzy set theory. Zadeh introduced fuzzy set theory, which is still being developed today. The following describes the background of fuzzy set theory as well as the specifics of the suggested fuzzy set theory with the golden ratio that is based on quantum mechanics.

Quantum mechanics is one of the branches of physics. This branch of physics includes sub-subatomic level, including concepts such as wave functions, quantum states, and operators [34]. The uncertainty of status of a massless particle is defined with the wave function $(\varphi)$. Fuzzy set theory is a way to express mathematical uncertainty. Recent papers highlight the similarities between these two subjects, despite the literature's inadequate treatment of them together. Fuzzy set theory and quantum physics are combined in the suggested model [35]. In quantum mechanics, the probability of a massless particle is equal the square of $\varphi$. These functions detailed in Equations (11)–(13) consist of a complex structure containing amplitude and phase angle $(\theta)$.

$$Q(|u>)=\varphi e^{j\theta} \tag{11}$$

$$|C>=\{|u_1>,|u_2>,\ldots,|u_n>\} \tag{12}$$

$$\sum_{|u>\subseteq|C>}|Q(|u>)|=1 \tag{13}$$

Where, C equals the collection of exhaustive events and $|\varphi_1|^2$ means the degree of belief. A conventional fuzzy set theory in Equation (14), has degree of a membership $(\mu_A)$. On the other hand, intuitionistic fuzzy sets (IFS) in Equation (15) have non-membership $(\nu_A)$ and $\mu_A$ functions.

$$A=\{\langle x,\mu_A(x)\rangle|x\in X\} \tag{14}$$

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\} \tag{15}$$

By adding degrees of neutral $(n_A)$ and refusal $(h_A)$ to IFS, picture fuzzy set (PFS) in Equation (16) obtained.

$$A = \{\langle x, \mu_A(x), n_A(x), v_A(x), h_A(x)\rangle | x \in X\} \tag{16}$$

Some operations computed with two PFSs are shown in Equations (17)–(21).

$$A \subseteq B \text{ if } \mu_A(x) \leq \mu_B(x) \text{ and } n_A(x) \leq n_B(x) \text{ and } v_A(x) \geq v_B(x), \forall x \in X \tag{17}$$

$$A = B \text{ if } A \subseteq B \text{ and } B \subseteq A \tag{18}$$

$$A \cup B = \{(x, max(\mu_A(x), \mu_B(x)), min(n_A(x), n_B(x)), min(v_A(x), v_B(x))) | x \in X\} \tag{19}$$

$$A \cap B = \{(x, min(\mu_A(x), \mu_B(x)), min(n_A(x), n_B(x)), max(v_A(x), v_B(x))) | x \in X\} \tag{20}$$

$$coA = \overline{A} = \{(x, v_A(x), n_A(x), \mu_A(x)) | x \in X\} \tag{21}$$

The rough number includes lower $\left(\underline{Apr}(C_i)\right)$-upper $(\overline{Apr}(C_i))$ approximation and rough boundary intervals $(Bnd(C_i))$. Using Equations (22)–(24), the relevant values of $C_i$ are defined.

$$\underline{Apr}(C_i) = \cup\left\{Y \in \frac{X}{R(Y)} \leq C_i\right\} \tag{22}$$

$$\overline{Apr}(C_i) = \cup\left\{Y \in \frac{X}{R(Y)} \geq C_i\right\} \tag{23}$$

$$Bnd(C_i) = \cup\left\{Y \in \frac{X}{R(Y)} \neq C_i\right\} \tag{24}$$

Lower $(\underline{Lim}(C_i))$, upper $(\overline{Lim}(C_i))$ limits and the rough number $(RN(C_i))$ of $C_i$ are shown with the help of Equations (25)–(27).

$$\underline{Lim}(C_i) = \sqrt[N_L]{\prod\nolimits_{i=1}^{N_L} Y \in \underline{Apr}(C_i)} \tag{25}$$

$$\overline{Lim}(C_i) = \sqrt[N_U]{\prod\nolimits_{i=1}^{N_U} Y \in \overline{Apr}(C_i)} \tag{26}$$

$$RN(C_i) = \lceil \underline{Lim}(C_i), \overline{Lim}(C_i) \rfloor \tag{27}$$

Where, $N_L$ and $N_U$ are numbers of objects for $\underline{Apr}(C_i)$ and $\overline{Apr}(C_i)$. Quantum picture fuzzy rough sets (QPFRS) are sets of PFS using quantum mechanics and rough number by the golden ratio. QPFRS is proposed to model different types of experts and results of analysis that are very close to reality. QPFRS in Equation (28) has membership $(C_{i\mu_A})$, neutral $(C_{in_A})$, non-membership $(C_{iv_A})$ and refusal $(C_{ih_A})$ functions.

$$|C_A> = \left\{ \begin{array}{c} \langle u, \left(\lceil \underline{Lim}(C_{i\mu_A}), \overline{Lim}(C_{i\mu_A}) \rfloor (u), \lceil \underline{Lim}(C_{in_A}), \overline{Lim}(C_{in_A}) \rfloor (u), \right. \\ \lceil \underline{Lim}(C_{iv_A}), \overline{Lim}(C_{iv_A}) \rfloor (u), \lceil \underline{Lim}(C_{ih_A}), \overline{Lim}(C_{ih_A}) \rfloor (u)) | u \in 2^{|C_A>} \end{array} \right\} \tag{28}$$

The components of PFRS are defined in Equations (29)–(44).

$$\underline{Lim}(C_{i\mu_A}) = \frac{1}{N_{L\mu_A}} \sum_{i=1}^{N_{L\mu_A}} Y \in \underline{Apr}(C_{i\mu_A}) \tag{29}$$

$$\underline{Lim}(C_{in_A}) = \frac{1}{N_{Ln_A}} \sum_{i=1}^{N_{Ln_A}} Y \in \underline{Apr}(C_{in_A}) \tag{30}$$

$$\underline{Lim}(C_{iv_A}) = \frac{1}{N_{Lv_A}} \sum_{i=1}^{N_{Lv_A}} Y \in \underline{Apr}(C_{iv_A}) \tag{31}$$

$$\underline{Lim}(C_{ih_A}) = \frac{1}{N_{L\pi_A}} \sum_{i=1}^{N_{L\pi_A}} Y \in \underline{Apr}(C_{ih_A}) \tag{32}$$

$$\overline{Lim}(C_{i\mu_A}) = \frac{1}{N_{U\mu_A}} \sum_{i=1}^{N_{U\mu_A}} Y \in \overline{Apr}(C_{i\mu_A}) \tag{33}$$

$$\overline{Lim}(C_{in_A}) = \frac{1}{N_{Un_A}} \sum_{i=1}^{N_{Un_A}} Y \in \overline{Apr}(C_{in_A}) \tag{34}$$

$$\overline{Lim}(C_{iv_A}) = \frac{1}{N_{Uv_A}} \sum_{i=1}^{N_{Uv_A}} Y \in \overline{Apr}(C_{iv_A}) \tag{35}$$

$$\overline{Lim}(C_{ih_A}) = \frac{1}{N_{U\pi_A}} \sum_{i=1}^{N_{U\pi_A}} Y \in \overline{Apr}(C_{ih_A}) \tag{36}$$

$$\underline{Apr}(C_{i\mu_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{i\mu_A}\right\} \tag{37}$$

$$\underline{Apr}(C_{in_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{in_A}\right\} \tag{38}$$

$$\underline{Apr}(C_{iv_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{iv_A}\right\} \tag{39}$$

$$\underline{Apr}(C_{ih_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{ih_A}\right\} \tag{40}$$

$$\overline{Apr}(C_{i\mu_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{i\mu_A}\right\} \tag{41}$$

$$\overline{Apr}(C_{in_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{in_A}\right\} \tag{42}$$

$$\overline{Apr}(C_{iv_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{iv_A}\right\} \tag{43}$$

$$\overline{Apr}(C_{ih_A}) = \cup\left\{Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{ih_A}\right\} \tag{44}$$

QPFRS, generated with combination of definitions, is shown using Equations (45) and (46).

$$C = \left[C_\mu . e^{j2\pi.\alpha}, C_n . e^{j2\pi.\gamma}, C_v . e^{j2\pi.\beta}, C_h . e^{j2\pi.T}\right] \tag{45}$$

$$\varphi^2 = |C_\mu(|u_i>)| \tag{46}$$

By adding the mathematical constant golden ratio (G) to the QPFRS, the expression in Equations (47) and (48) is computed. Approximately, G equals $(1 + \sqrt{5})/2$..

$$C_n = \frac{C_\mu}{G} \tag{47}$$

$$C_h = \frac{C_v}{G} \tag{48}$$

The phase angle of the membership function for the probability of event $|u_i>$ in the realm of QPFS is symbolized with α in Equations (49)–(51).

$$\alpha = |C_\mu(|u_i>)| \tag{49}$$

$$\gamma = \frac{\alpha}{G} \tag{50}$$

$$T\ (32) = \frac{\beta}{G} \tag{51}$$

Some operations definitions for the two QPFRS are presented in Equations (52)–(55). $\lambda$ is non-negative number.

### 3.4. M-SWARA with QPFRS

When using optimization techniques, weighing criteria is crucial. The SWARA approach is a multi-criteria decision-making technique that uses progressive weighting. This method led to the development of the Multi SWARA (M-SWARA) method, a multi-criterion weighing technique. Below is a description of each of the eight M-SWARA technique steps [36].

In *Step 9,* criteria for global energy transition in hydrogen production

$$\lambda * \widetilde{A}_c = \left\{ \begin{array}{l} \left[ \underline{Lim}(C_{\mu_{\tilde{A}}})\lambda, \overline{Lim}(C_{\mu_{\tilde{A}}})\lambda \right] e^{j2\pi.\left[\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)\lambda,\ \left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)\lambda\right]}, \left[ \underline{Lim}(C_{n_{\tilde{A}}})\lambda, \overline{Lim}(C_{n_{\tilde{A}}})\lambda \right] e^{j2\pi.\left[\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)\lambda,\ \left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)\lambda\right]}, \\ \left[ \underline{Lim}(C_{v_{\tilde{A}}})\lambda, \overline{Lim}(C_{v_{\tilde{A}}})\lambda \right] e^{j2\pi.\left[\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)\lambda,\ \left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)\lambda\right]}, \left[ \underline{Lim}(C_{h_{\tilde{A}}})\lambda, \overline{Lim}(C_{h_{\tilde{A}}})\lambda \right] e^{j2\pi.\left[\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)\lambda,\ \left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)\lambda\right]} \end{array} \right\} \tag{52}$$

$$\widetilde{A}_c^{\ \lambda} = \left\{ \begin{array}{l} \left[ \underline{Lim}(C_{\mu_{\tilde{A}}})^{\lambda}, \overline{Lim}(C_{\mu_{\tilde{A}}})^{\lambda} \right] e^{j2\pi.\left[\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)^{\lambda},\left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \left[ \underline{Lim}(C_{n_{\tilde{A}}})^{\lambda}, \overline{Lim}(C_{n_{\tilde{A}}})^{\lambda} \right] e^{j2\pi.\left[\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)^{\lambda},\left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \\ \left[ \underline{Lim}(C_{v_{\tilde{A}}})^{\lambda}, \overline{Lim}(C_{v_{\tilde{A}}})^{\lambda} \right] e^{j2\pi.\left[\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)^{\lambda},\left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \left[ \underline{Lim}(C_{h_{\tilde{A}}})^{\lambda}, \overline{Lim}(C_{h_{\tilde{A}}})^{\lambda} \right] e^{j2\pi.\left[\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)^{\lambda},\left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \end{array} \right\} \tag{53}$$

$$\widetilde{A}_c \cup \widetilde{B}_c = \left\{ \begin{array}{l} \left[ min\left( \underline{Lim}(C_{\mu_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{\mu_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{\alpha}_{\tilde{B}}}{2\pi}\right)} \right), max\left( \overline{Lim}(C_{\mu_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{\mu_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[ min\left( \underline{Lim}(C_{n_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{n_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{\gamma}_{\tilde{B}}}{2\pi}\right)} \right), max\left( \overline{Lim}(C_{n_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{n_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[ min\left( \underline{Lim}(C_{v_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{v_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{\beta}_{\tilde{B}}}{2\pi}\right)} \right), max\left( \overline{Lim}(C_{v_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{v_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[ min\left( \underline{Lim}(C_{h_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{h_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{T}_{\tilde{B}}}{2\pi}\right)} \right), max\left( \overline{Lim}(C_{h_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{h_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{T}_{\tilde{B}}}{2\pi}\right)} \right) \right] \end{array} \right\} \tag{54}$$

$$\widetilde{A}_c \cap \widetilde{B}_c = \left\{ \begin{array}{l} \left[ max\left( \underline{Lim}(C_{\mu_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{\mu_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{\alpha}_{\tilde{B}}}{2\pi}\right)} \right), min\left( \overline{Lim}(C_{\mu_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{\mu_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[ max\left( \underline{Lim}(C_{n_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{n_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{\gamma}_{\tilde{B}}}{2\pi}\right)} \right), min\left( \overline{Lim}(C_{n_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{n_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[ max\left( \underline{Lim}(C_{v_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{v_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{\beta}_{\tilde{B}}}{2\pi}\right)} \right), min\left( \overline{Lim}(C_{v_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{v_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[ max\left( \underline{Lim}(C_{h_{\tilde{A}}})e^{j2\pi.\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}(C_{h_{\tilde{B}}})e^{j2\pi.\left(\frac{\underline{T}_{\tilde{B}}}{2\pi}\right)} \right), min\left( \overline{Lim}(C_{h_{\tilde{A}}})e^{j2\pi.\left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}(C_{h_{\tilde{B}}})e^{j2\pi.\left(\frac{\overline{T}_{\tilde{B}}}{2\pi}\right)} \right) \right] \end{array} \right\} \tag{55}$$

is defined. In result of literature review, a set of criteria is obtained. *Step 10* involves constructing computed linguistic evaluations of the decision makers for the criteria. In this step, operations of QPFRS in Section 3.3 is taken into consideration. *Step 11* is about obtaining QPFN in Equation (56) for the relationship matrix (C $= \left[C_{ij}\right]_{n \times n}$). Where, k is number of decision makers and QPFR direct relation matrix is symbolized by C.

$$C_k = \begin{bmatrix} 0 & C_{12} & \cdots & \cdots & C_{1n} \\ C_{21} & 0 & \cdots & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & \cdots & 0 \end{bmatrix} \tag{56}$$

*Step 12* covers defining of expert weighted QPFR for the relationship matrix with the help of Equations (57) and (58). $w_k$ represents the weights of decision makers.

$$w_k \times \mathrm{C} \tag{57}$$

$$\mathrm{C} = \begin{pmatrix} \left[min_{i=1}^{k}\left(\underline{Lim}\left(C_{\mu_{ij}}\right)\right), max_{i=1}^{k}\left(\overline{Lim}\left(C_{\mu_{ij}}\right)\right)\right] e^{j2\pi.\left[min_{i=1}^{k}\left(\frac{\underline{\alpha}_{ij}}{2\pi}\right), max_{i=1}^{k}\left(\frac{\overline{\alpha}_{ij}}{2\pi}\right)\right]}, \\ \left[min_{i=1}^{k}\left(\underline{Lim}\left(C_{n_{ij}}\right)\right), max_{i=1}^{k}\left(\overline{Lim}\left(C_{n_{ij}}\right)\right)\right] e^{j2\pi.\left[min_{i=1}^{k}\left(\frac{\underline{\gamma}_{ij}}{2\pi}\right), max_{i=1}^{k}\left(\frac{\overline{\gamma}_{ij}}{2\pi}\right)\right]}, \\ \left[min_{i=1}^{k}\left(\underline{Lim}\left(C_{\nu_{ij}}\right)\right), max_{i=1}^{k}\left(\overline{Lim}\left(C_{\nu_{ij}}\right)\right)\right] e^{j2\pi.\left[min_{i=1}^{k}\left(\frac{\underline{\beta}_{ij}}{2\pi}\right), max_{i=1}^{k}\left(\frac{\overline{\beta}_{ij}}{2\pi}\right)\right]}, \\ \left[min_{i=1}^{k}\left(\underline{Lim}\left(C_{h_{ij}}\right)\right), max_{i=1}^{k}\left(\overline{Lim}\left(C_{h_{ij}}\right)\right)\right] e^{j2\pi.\left[min_{i=1}^{k}\left(\frac{\underline{T}_{ij}}{2\pi}\right), max_{i=1}^{k}\left(\frac{\overline{T}_{ij}}{2\pi}\right)\right]} \end{pmatrix} \tag{58}$$

*Step 13* is about computing of defuzzified values the criteria using Equation (59).

$$Defc_i = \frac{\begin{pmatrix} \underline{Lim}(C_{\mu_i}) - \underline{Lim}(C_{n_i}) + \underline{Lim}(C_{\mu_i}).(\underline{Lim}(C_{\nu_i}) - \underline{Lim}(C_{h_i})) + \left(\frac{\underline{\alpha}_{ij}}{2\pi}\right) - \left(\frac{\underline{\gamma}_{ij}}{2\pi}\right) + \left(\frac{\underline{\alpha}_{ij}}{2\pi}\right).\left(\left(\frac{\underline{\beta}_{ij}}{2\pi}\right) - \left(\frac{\underline{T}_{ij}}{2\pi}\right)\right) + \\ \overline{Lim}(C_{\mu_i}) - \overline{Lim}(C_{n_i}) + \overline{Lim}(C_{\mu_i}).(\overline{Lim}(C_{\nu_i}) - \overline{Lim}(C_{h_i})) + \left(\frac{\overline{\alpha}_{ij}}{2\pi}\right) - \left(\frac{\overline{\gamma}_{ij}}{2\pi}\right) + \left(\frac{\overline{\alpha}_{ij}}{2\pi}\right).\left(\left(\frac{\overline{\beta}_{ij}}{2\pi}\right) - \left(\frac{\overline{T}_{ij}}{2\pi}\right)\right) \end{pmatrix}}{2} \tag{59}$$

In *Step 14,* the relation matrix is normalized. *Step 15* includes calculating of the comparative importance (sj), coefficient value (kj), recalculated weight (qj) and weights of the criteria (wj) for the relationship degrees of each criterion with the help of Equations (60)–(62).

$$k_j = \begin{cases} 1\; j = 1 \\ s_j + 1\; j > 1 \end{cases} \tag{60}$$

$$q_j = \begin{cases} 1\; j = 1 \\ \dfrac{q_{j-1}}{k_j}\; j > 1 \end{cases} \quad If\ s_{j-1} = s_j, q_{j-1} = q_j \quad If\ s_j = 0, k_{j-1} = k_j \tag{61}$$

$$w_j = \frac{q_j}{\sum\limits_{k=1}^{n} q_k} \tag{62}$$

Stable values are calculated using powers of 2t+1. The t is the biggest value. *Step 16* covers constructing of the relation matrix and the direction among the criteria. The threshold value equals the mean of elements of the relationship matrix. The criterion above threshold is defined as influencing.

### 3.5. VIKOR with QPFRS

One of the multi-criteria decision-making ranking methods is VIKOR. The VIKOR method is a consensus-based method that serves this purpose. The equations and explanations of the six-step model are as below [37].

*Step 17* involves constructing the completed linguistic evaluations of decision makers for the alternatives. In *Step 18,* the QPFN for the decision matrix shown in Equation (63) is constructed.

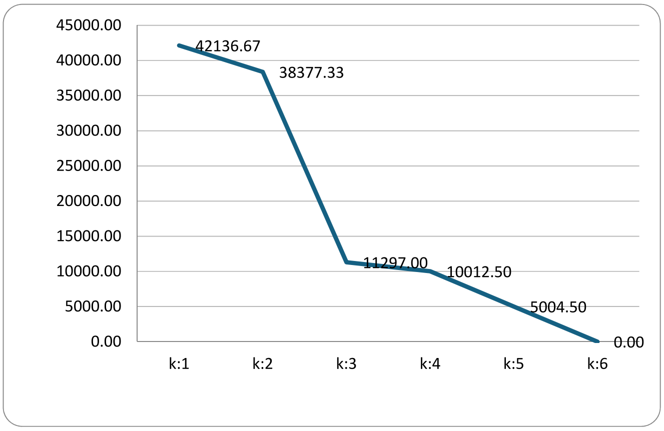

**Fig. 2.** The plot of the WCSS values and k numbers.

**Table 1**
Specifications of the DMs.

| Decision Maker | Education | Experience (yy) | Salary ($) | Age |
|---|---|---|---|---|
| DM1 | PhD | 16 | 2400 | 42 |
| DM2 | Master | 14 | 2350 | 40 |
| DM3 | Master | 15 | 2500 | 44 |
| DM4 | Bachelor | 18 | 2400 | 48 |
| DM5 | PhD | 15 | 2600 | 46 |
| DM6 | Bachelor | 16 | 2500 | 44 |

$$X_k = \begin{bmatrix} 0 & X_{12} & \cdots & \cdots & X_{1m} \\ X_{21} & 0 & \cdots & \cdots & X_{2m} \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & \cdots & 0 \end{bmatrix} \tag{63}$$

*Step 19* covers determining the expert weighted QPFRS for the decision matrix. To this end, the procedures of QPFRS in Section 3.3 are taken into consideration. *Step 20* involves computing defuzzified decision values using Equation (59). With *Step 21*, mean group utility (Si), maximal regret (Ri) and final ranking (Qi) are constructed. The best $\widetilde{f}_J^*$ and worst $\widetilde{f}_j^-$ values for each criterion are found by Equation (64). Then, Si, Ri and Qi values are calculated by Equations (65)–(67).

$$\widetilde{f}_J^* = \max_i \widetilde{x}_{ij}, \text{ and } \widetilde{f}_j^- = \min_i \widetilde{x}_{ij} \tag{64}$$

$$\widetilde{S}_i = \sum_{i=1}^{n} \widetilde{w}_j \frac{\left(\left|\widetilde{f}_j^* - \widetilde{x}_{ij}\right|\right)}{\left(\left|\widetilde{f}_j^* - \widetilde{f}_j^-\right|\right)} \tag{65}$$

$$\widetilde{R}_i = \max_j \left[\widetilde{w}_j \frac{\left(\left|\widetilde{f}_j^* - \widetilde{x}_{ij}\right|\right)}{\left(\left|\widetilde{f}_j^* - \widetilde{f}_j^-\right|\right)}\right] \tag{66}$$

**Table 2**
The DMs' weights with pareto principle.

| Decision Makers | Weights | Normalized weights with pareto principle |
|---|---|---|
| DM1 | 0.03 | 0.10 |
| DM2 | 0.00 | 0.00 |
| DM3 | 0.31 | 0.27 |
| DM4 | 0.03 | 0.10 |
| DM5 | 0.31 | 0.27 |
| DM6 | 0.31 | 0.27 |

**Table 3**
Criteria set for global energy transition in hydrogen production.

| Criteria | Codes |
|---|---|
| Decarbonization | DECARB |
| Renewable Energy Expansion | REENEX |
| Energy Efficiency | ENEFF |
| Sustainable Development | SUDEV |

Method alternatives for hydrogen production are coded in Table 4.

$$\widetilde{Q}_i = v(\widetilde{S}_i - \widetilde{S}^*) / (\widetilde{S}^- - \widetilde{S}^*) + (1 - v)(\widetilde{R}_i - \widetilde{R}^*) / (\widetilde{R}^- - \widetilde{R}^*) \tag{67}$$

With these values, two assumptions are tested. The first assumption is given with Equation (68). The second assumption included ordering of S and R values.

$$Q(A^{(2)}) - Q(A^{(1)}) \geq \frac{1}{(j - 1)} \tag{68}$$

In *Step 22,* comparative ranking values are calculated with sensitivity analysis.

## 4. Analysis results

The analysis result of the stages displayed in Fig. 1 is presented in this section.

### 4.1. Prioritizing the experts with AI-based decision-making method

For *Step 1*, the specifications of the decision makers are presented in Table 1.

*Step 2* is about computing the optimal k for clustering the DMs. The WCSS are obtained with Equation (1). With k is between 1 and 6, WCSSs are given in Ta ble A1 in Appendix. The plot drawn for the Elbow method is illustrated in Fig. 2. According to Fig. 2, the optimal value of k, elbow point, is 3.

At the end of *Step 3* process, the k-means clustering algorithm for clustering DMs is applied with the help of Equations (2) and (3). According to Ta ble A2, DM1 and DM4 are experts in the first cluster, while DM2 is the second cluster. DM3, DM5 and DM6 are stated in third

**Table 4**
Method alternatives for hydrogen production.

| Alternatives | Codes |
|---|---|
| Thermal processes including steam methane reforming and biomass gasification | A1 |
| Electrolytic processes splitting water into hydrogen and oxygens with water electrolysis | A2 |
| Photolytic Processes using photobiological facilities and photoelectrochemical cells | A3 |
| Methane pyrolysis and thermochemical processes using solar energy to produce hydrogen from water or hydrocarbons | A4 |

**Table 5**
Stable matrix.

| | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| DECARB | 0.247 | 0.247 | 0.247 | 0.247 |
| REENEX | 0.252 | 0.252 | 0.252 | 0.252 |
| ENEFF | 0.251 | 0.251 | 0.251 | 0.251 |
| SUDEV | 0.250 | 0.250 | 0.250 | 0.250 |

In respect to Table 5, the most important criteria is Renewable Energy Expansion because of highest value. Energy Efficiency are second important criteria. The weight of Sustainable Development is 0.25. Decarbonization is last criteria with a weight of 0.247.

**Table 6**
Completed linguistic evaluations of decision makers for the alternatives.

| DM1 | | | | |
|---|---|---|---|---|
| | DECARB | REENEX | ENEFF | SUDEV |
| A1 | G | B | B | B |
| A2 | G | G | B | B |
| A3 | G | F | G | G |
| A4 | F | G | B | F |
| **DM3** | | | | |
| A1 | B | F | G | G |
| A2 | G | F | B | G |
| A3 | B | F | B | G |
| A4 | F | G | B | B |
| **DM4** | | | | |
| A1 | G | B | F | G |
| A2 | F | B | P | G |
| A3 | B | F | P | G |
| A4 | B | G | F | G |
| **DM5** | | | | |
| A1 | G | F | B | B |
| A2 | B | F | B | B |
| A3 | B | G | B | B |
| A4 | G | G | G | B |
| **DM6** | | | | |
| A1 | B | G | G | G |
| A2 | B | G | B | B |
| A3 | B | G | B | B |
| A4 | G | B | B | B |

cluster. As a result of *Step 4,* the weights of the experts by considering the cluster weights of the experts are computed by Equations (4)–(7). The mean standard deviations are illustrated in Table A3. Using Equation (8), Table is given information about the weights of the DMs.

According to Table 2, DM3, DM5, and DM6 have the first priorities because they have the high stature as 0.31. The normalized weights of the DMs are also calculated with the pareto principle to explore all impacts of the decision makers together. The Pareto Principle, also known as the 80/20 rule, is a powerful concept used in various fields, including decision-making and resource allocation. It states that for many phenomena, about 80% of the consequences are produced by 20% of the causes. Accordingly, the Pareto Principle helps identify the most significant factors in a set of data. For instance, it can help determine which experts (the 20%) are contributing to the majority (80%) of the expert choices. When it comes to computing the relative importance of the experts among them, the Pareto Principle can be particularly useful. The normalized weights will be properly considered for weighting and ranking the factors in the following stages. According to Table 2, DM1, DM3, DM4, DM5 and DM6 have the priorities in the expert team. So, the evaluations of these 5 decision makers except DM2 are considered only to assess the criteria and alternatives.

### 4.2. *Estimate the missing evaluations of hydrogen production methods in the global energy transition with expert recommender system*

As part of *Step 5*, the criteria for global energy transition in hydrogen production are coded in Table 3.

**Table 7**
Comparative ranking values with sensitivity analysis.

| Extended VIKOR (v:.5) | | | | |
|---|---|---|---|---|
| Alternatives | Case 1 | Case 2 | Case 3 | Case 4 |
| A1 | 1 | 1 | 1 | 1 |
| A2 | 3 | 3 | 3 | 4 |
| A3 | 4 | 4 | 4 | 3 |
| A4 | 2 | 2 | 2 | 2 |
| **Extended TOPSIS** | | | | |
| A1 | 1 | 1 | 1 | 1 |
| A2 | 3 | 3 | 3 | 3 |
| A3 | 4 | 4 | 4 | 4 |
| A4 | 2 | 2 | 2 | 2 |

With *Step 6*, linguistic expressions for the dataset are given in Table A4, A5 and A6. For *Step 7*, similarity degrees of DMs are computed using Equation (9). The results for the criteria and alternative are shown in Table A7 and Table A8. As part of *Step 8*, undefined expressions are calculated by Equation (10). The results for the alternatives and criteria are presented in Tables A9 and A10, respectively.

### 4.3. *Weighting the criteria for global energy transition in hydrogen production with QPFR-M-SWARA*

As a result of *Step 9*, criteria for global energy transition in hydrogen production are defined, shown in Table 3. For *Step 10,* the completed linguistic evaluations of DMs for the criteria in Table A11 are constructed. *Step 11* covers obtaining QPFN for relation matrix in Table A12. In process of *Step 12*, expert weighted QPFRS is defined with Equations (57) and (58). The result is presented in Table A13.

*Step 13* is about computing defuzzified values for the criteria with the help of Equation (59). The result values are illustrated in Table A14. As result of *Step 14*, the relation matrix is normalized. The results are given in Table A15. For *Step 15*, the values of s, k, q and w calculated with the help of Equations (60)–(62) are displayed in Table A16. In *Step 16*, the relation matrix is constructed. The results are shared in Table A17. The stable matrix is illustrated in Table 5.

### 4.4. *Ranking the method alternatives for hydrogen production with QPFR-VIKOR*

*Step 17* is about constructing the completed linguistic opinions of the decision makers, taking into account the alternatives in Table 4. These evaluations are illustrated in Table 6.

*Step 18* covers constructing the QPFN for decision matrix in Table A18 with Equation (63). For *Step 19*, expert weighted QPFRS are determined for the decision matrix, depicted in Table A19. With *Step 20*, using Equation (59), the defuzzified decision values in Table A20 are computed.

S, R and Q is computed with Equations (64)–(67) in process of *Step 21*. The results of this operation are depicted in Table A21 is given in the appendix. With *Step 22*, the comparative ranking values with sensitivity analysis is calculated. TOPSIS is preferred as the comparison method. The results of eight-cases are displayed in Table 7.

Thermal processes including steam methane reforming and biomass gasification are obtained as the most suitable alternative of strategies for hydrogen production. Methane pyrolysis and thermochemical processes using solar energy to produce hydrogen from water or hydrocarbons is in second rank. According to Table 7, this ranking of alternatives for hydrogen production is the same in four cases of two analysis results. Therefore, it is concluded that the results are consistent.

## 5. Discussion

In hydrogen production, renewable energy expansion is essential for the success of the global energy transition. Renewable energy expansion enables the increase of clean energy sources used in hydrogen production. In this context, clean energy sources should be preferred instead of fossil fuels in the process of obtaining hydrogen. Moon et al. [38] discussed that to achieve this goal, necessary measures must be taken to increase the use of renewable energy. In this context, it is important to provide some financial support by states. For example, providing tax deductions significantly increases the cost-effectiveness of these projects [39]. This situation supports more investors to focus on this area. On the other hand, Bouzgarrou et al. [40] defined that it is also necessary to conduct research and development studies on renewable energy technology. Karayel and Dincer [41] defined that thanks to these studies, it is possible to reduce the costs of renewable energy projects. This allows these projects to increase their competitiveness compared to fossil fuels. In addition to this issue, energy efficiency in hydrogen production is extremely important for the global energy transition. Energy efficiency enables more efficient use of energy resources used in hydrogen production. Mendrela et al. [42] indicated that this contributes significantly to reducing hydrogen production costs. Cai et al. [43] concluded that energy efficiency can reduce carbon emissions in hydrogen production processes. This helps the business' operational processes to cause less environmental damage.

Thermal processes such as steam methane reforming and biomass gasification is found as the most optimal alternative. This investment alternative provides some significant advantages. Zou et al. [44] stated that steam methane reforming is accepted as an efficient process to produce hydrogen. With the help of this advanced technology, high amount of hydrogen can be obtained. Tan et al. [45] discussed that this situation provides an important cost advantages to the companies. In addition to this issue, steam methane reforming has a scalability advantage so that it can be possible to meet varying demands for hydrogen production. Moreover, owing to the biomass gasification, waste reduction can be more possible. This situation has a powerful contribution to minimize carbon emission in the hydrogen generation process. On the other side, methane pyrolysis and thermochemical processes utilizing solar energy for hydrogen production offer some benefits for sustainable hydrogen production. This condition provides some opportunities to increase renewable energy usage [46]. In other words, carbon free hydrogen production process can be implemented [47].

## 6. Conclusion

In this article, it is aimed to examine hydrogen production methods for global energy transition. Within this context, a four-stage model has been constructed. Firstly, AI-based decision-making can be implemented for expert prioritization. In the second stage, recommender system is conducted with collaborative filtering to complete the missing evaluations. Thirdly, selected criteria are weighted by using M-SWARA with QPFRS. Finally, method alternatives for hydrogen production are ranked via QPFRS adopted VIKOR. It is identified that renewable energy expansion and energy efficiency are the most important criteria for global energy transition in hydrogen production. On the other side, the ranking results denote that thermal processes including steam methane reforming and biomass gasification is the most appropriate method alternatives for hydrogen production.

Ensuring efficient hydrogen production using renewable energy sources is vital for the global energy transition. It is possible to determine some policies to increase these energy projects. Renewable energy incentives play a very important role in this process. This can significantly reduce the costs of projects. Thus, investors will focus more on these projects whose profitability can be increased. Low-interest loans also allow investors to access the financing resources more easily. Similarly, increased research and development activities also support the development of renewable energy projects for efficient hydrogen production. In this context, a coordinated cooperation effort should be carried out between the private sector and universities. On the other hand, training programs should be organized to increase public awareness on these issues. This provides the opportunity to increase social acceptance for renewable energy projects.

The most significant contribution for doing this study is that AI technique is integrated into the model and experts' importance coefficients are can be computed. Furthermore, by using the collaborative filtering technique, empty evaluations can be filled scientifically. This situation contributes to the quality of the analysis process in many ways. Owing to this technique, experts are given the opportunity not to answer questions they are not very sure about. This proposed model is also applicable for other industries. The main purpose of all companies is to increase the profitability. In this process, these companies should give appropriate strategic decision while considering many different issues. Thus, this proposed model mainly helps these companies to reach this objective. The main limitation of this study is that only energy transition way of hydrogen production is taken into consideration. However, the effectiveness of the hydrogen storage process can be examined in the following studies. The proposed model has also some limitations. In the ranking process, VIKOR is taken into consideration. However, there are some criticisms regarding existing ranking approaches. Therefore, a novel ranking methodology should be proposed in the future studies.

## CRediT authorship contribution statement

**Hasan Dinçer:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Serhat Yüksel:** Writing – review & editing, Writing – original draft, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Serkan Eti:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Merve Acar:** Writing – original draft, Methodology, Formal analysis, Data curation.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix

**Table A1**
The set of WCSS values for different k values

| K = 1 | | K = 2 | | K = 3 | | K = 4 | | K = 5 | | K = 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS |
| DM1 | 3407.89 | DM1 | 112.44 | DM1 | 2502.00 | DM1 | 0 | DM1 | 0 | DM1 | 0 |
| DM2 | 11754.89 | DM4 | 1124.44 | DM6 | 2502.00 | **Cluster 2** | **WCSS** | **Cluster 2** | **WCSS** | **Cluster 2** | **WCSS** |
| DM3 | 1736.56 | DM6 | 4445.78 | **Cluster 2** | **WCSS** | DM2 | 0 | DM2 | 0 | DM2 | 0 |
| DM4 | 3425.22 | **Cluster 2** | **WCSS** | DM2 | 645.25 | **Cluster 3** | **WCSS** | **Cluster 3** | **WCSS** | **Cluster 3** | **WCSS** |
| DM5 | 20074.89 | DM2 | 17789.44 | DM4 | 645.25 | DM3 | 2501.25 | DM3 | 0 | DM3 | 0 |
| DM6 | 1737.22 | DM3 | 278.44 | **Cluster 3** | **WCSS** | DM5 | 2501.25 | **Cluster 4** | **WCSS** | **Cluster 4** | **WCSS** |

(*continued on next page*)

**Table A1** (*continued*)

| K = 1 | | K = 2 | | K = 3 | | K = 4 | | K = 5 | | K = 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS | Cluster 1 | WCSS |
| | | DM5 | 13618.78 | DM3 | 2501.25 | **Cluster 4** | **WCSS** | DM4 | 0 | DM4 | 0 |
| | | | | DM5 | 2501.25 | DM4 | 2505.00 | **Cluster 5** | **WCSS** | **Cluster 5** | **WCSS** |
| | | | | | | DM6 | 2505.00 | DM5 | 2502.25 | DM5 | 0 |
| | | | | | | | | DM6 | 2502.25 | **Cluster 6** | **WCSS** |
| **Total** | 42136.67 | **Total** | 38377.33 | **Total** | 11297.00 | **Total** | 10012.50 | **Total** | 5004.50 | DM6 | 0 |

**Table A2**
Iteration results of optimal cluster value

| Iteration (DM 1 is in Cluster 1; DM2 is in Cluster 2; DM 3 is in Cluster 3) | | | | |
|---|---|---|---|---|
| Initial Cluster Centers | | | | |
| Decision Maker | Distance to C1 | Distance to C2 | Distance to C3 | Cluster Assignment |
| DM1 | 0.00 | 5.09 | 10.03 | 1 |
| DM2 | 5.09 | 0.00 | 15.06 | 2 |
| DM3 | 10.03 | 15.06 | 0.00 | 3 |
| DM4 | 6.63 | 5.80 | 10.13 | 1 |
| DM5 | 20.04 | 25.08 | 10.02 | 3 |
| DM6 | 10.04 | 15.07 | 1.41 | 3 |
| **Average of Data Points** | | | | |
| DM1 | 3.32 | 5.09 | 133.37 | 1 |
| DM2 | 5.34 | 0.00 | 183.40 | 2 |
| DM3 | 10.02 | 15.06 | 33.34 | 3 |
| DM4 | 3.32 | 5.80 | 133.41 | 1 |
| DM5 | 20.01 | 25.08 | 66.69 | 3 |
| DM6 | 10.01 | 15.07 | 33.36 | 3 |

**Table A3**
The standard deviations of the features and the weights by clusters

| Cluster center | Size | Education | Experience | Salary | Age | Mean SD | Weight |
|---|---|---|---|---|---|---|---|
| C1 | 2 | 1 | 1 | 0 | 3 | 1.25 | 2.50 |
| C2 | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| C3 | 3 | 0.82 | 0.47 | 47.14 | 0.94 | 12.34 | 37.03 |

**Table A4**
Linguistic scales and quantum picture fuzzy numbers for evaluation

| Linguistic Scales for Criteria | Linguistic Scales for Alternatives | Recommender Degrees | Possibility Degrees | QPFNs |
|---|---|---|---|---|
| No influence (n) | Weakest (w) | 1 | 0.40 | $\begin{bmatrix}\sqrt{.16}e^{j2\pi..4},\\ \sqrt{.10}e^{j2\pi..25},\\ \sqrt{.46}e^{j2\pi..22},\\ \sqrt{.28}e^{j2\pi..13}\end{bmatrix}$ |
| somewhat influence (s) | Poor (p) | 2 | 0.45 | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ |
| medium influence (m) | Fair (f) | 3 | 0.50 | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ |
| high influence (h) | Good (g) | 4 | 0.55 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| very high influence (vh) | Best (b) | 5 | 0.60 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |

**Table A5**
Linguistic evaluations of the decision makers for the relation matrix

| | DM 1 | DM 2 | DM 3 | DM 4 | DM 5 |
|---|---|---|---|---|---|
| DECARB- REENEX | 4 | n/a | 4 | 5 | 3 |
| DECARB- ENEFF | 5 | 5 | 3 | 3 | 4 |
| DECARB- SUDEV | 3 | 4 | 5 | n/a | 4 |
| REENEX- DECARB | n/a | 3 | 3 | 5 | 4 |
| REENEX- ENEFF | 3 | 2 | n/a | 3 | 4 |
| REENEX- SUDEV | 5 | n/a | 4 | 3 | 5 |
| ENEFF- DECARB | 5 | 2 | 5 | 3 | 5 |
| ENEFF- REENEX | 5 | 4 | 3 | 3 | 5 |
| ENEFF- SUDEV | 5 | n/a | 4 | 5 | 4 |
| SUDEV- DECARB | 2 | 4 | 4 | n/a | 3 |
| SUDEV- REENEX | 3 | n/a | 4 | 4 | 3 |
| SUDEV- ENEFF | 2 | 5 | 4 | 4 | 2 |

**Table A6**
Linguistic evaluations of the decision makers for the decision matrix

| | DM 1 | DM 2 | DM 3 | DM 4 | DM 5 |
|---|---|---|---|---|---|
| DECARB- A1 | n/a | n/a | 4 | 4 | 5 |
| DECARB- A2 | 4 | 4 | 3 | 5 | 5 |
| DECARB- A3 | 4 | 5 | 5 | n/a | 5 |
| DECARB- A4 | n/a | 3 | 5 | 4 | 4 |
| REENEX- A1 | 5 | 3 | 5 | 3 | 4 |
| REENEX- A2 | 4 | 3 | 5 | 3 | 4 |
| REENEX- A3 | 3 | 3 | n/a | 4 | 4 |
| REENEX- A4 | n/a | 4 | 4 | 4 | 5 |
| ENEFF- A1 | 5 | n/a | 3 | 5 | 4 |
| ENEFF- A2 | n/a | 5 | 2 | 5 | 5 |
| ENEFF- A3 | 4 | 5 | 2 | 5 | n/a |
| ENEFF- A4 | 5 | n/a | 3 | 4 | 5 |
| SUDEV- A1 | 5 | 4 | 4 | 5 | n/a |
| SUDEV- A2 | 5 | 4 | 4 | 5 | 5 |
| SUDEV- A3 | 4 | 4 | n/a | 5 | 5 |
| SUDEV- A4 | 3 | n/a | 4 | 5 | 5 |

**Table A7**
Similarity index matrix of the decision makers for the criteria

| | DM 1 | DM 2 | DM 3 | DM 4 | DM 5 |
|---|---|---|---|---|---|
| DM 1 | 1.00 | −0.16 | −0.22 | −0.19 | 0.78 |
| DM 2 | −0.16 | 1.00 | −0.32 | 0.08 | −0.45 |
| DM 3 | −0.22 | −0.32 | 1.00 | −0.05 | −0.03 |
| DM 4 | −0.19 | 0.08 | −0.05 | 1.00 | −0.49 |
| DM 5 | 0.78 | −0.45 | −0.03 | −0.49 | 1.00 |

**Table A8**
Similarity index matrix of the decision makers for the alternatives

| | DM 1 | DM 2 | DM 3 | DM 4 | DM 5 |
|---|---|---|---|---|---|
| DM 1 | 1.00 | 0.03 | −0.02 | −0.05 | −0.04 |
| DM 2 | 0.03 | 1.00 | −0.59 | 0.66 | 0.69 |
| DM 3 | −0.02 | −0.59 | 1.00 | −0.62 | −0.34 |
| DM 4 | −0.05 | 0.66 | −0.62 | 1.00 | 0.52 |
| DM 5 | −0.04 | 0.69 | −0.34 | 0.52 | 1.00 |

**Table A9**
Iterative completion of missing expressions for the criteria

| | DM 1 | DM 3 | DM 4 | DM 5 | DM 6 |
|---|---|---|---|---|---|
| DECARB- REENEX | 4 | 5 (Iteration 1) | 4 | 5 | 3 |
| DECARB- ENEFF | 5 | 5 | 3 | 3 | 4 |
| DECARB- SUDEV | 3 | 4 | 5 | 4 (Iteration 1) | 4 |
| REENEX- DECARB | 4 (Iteration 1) | 3 | 3 | 5 | 4 |
| REENEX- ENEFF | 3 | 2 | 4 (Iteration 1) | 3 | 4 |
| REENEX- SUDEV | 5 | 3 (Iteration 1) | 4 | 3 | 5 |
| ENEFF- DECARB | 5 | 2 | 5 | 3 | 5 |
| ENEFF- REENEX | 5 | 4 | 3 | 3 | 5 |
| ENEFF- SUDEV | 5 | 5 (Iteration 1) | 4 | 5 | 4 |
| SUDEV- DECARB | 2 | 4 | 4 | 4 (Iteration 1) | 3 |
| SUDEV- REENEX | 3 | 4 (Iteration 1) | 4 | 4 | 3 |
| SUDEV- ENEFF | 2 | 5 | 4 | 4 | 2 |

**Table A10**
Iterative completion of missing expressions for the alternatives

| | DM 1 | DM 3 | DM 4 | DM 5 | DM 6 |
|---|---|---|---|---|---|
| DECARB- A1 | 4 (Iteration 2) | 5 (Iteration 1) | 4 | 4 | 5 |
| DECARB- A2 | 4 | 4 | 3 | 5 | 5 |
| DECARB- A3 | 4 | 5 | 5 | 5 (Iteration 1) | 5 |
| DECARB- A4 | 3 (Iteration 1) | 3 | 5 | 4 | 4 |
| REENEX- A1 | 5 | 3 | 5 | 3 | 4 |
| REENEX- A2 | 4 | 3 | 5 | 3 | 4 |
| REENEX- A3 | 3 | 3 | 3 (Iteration 1) | 4 | 4 |
| REENEX- A4 | 4 (Iteration 1) | 4 | 4 | 4 | 5 |
| ENEFF- A1 | 5 | 4 (Iteration 1) | 3 | 5 | 4 |
| ENEFF- A2 | 5 (Iteration 1) | 5 | 2 | 5 | 5 |
| ENEFF- A3 | 4 | 5 | 2 | 5 | 5 (Iteration 1) |
| ENEFF- A4 | 5 | 5 (Iteration 1) | 3 | 4 | 5 |
| SUDEV- A1 | 5 | 4 | 4 | 5 | 4 (Iteration 1) |
| SUDEV- A2 | 5 | 4 | 4 | 5 | 5 |
| SUDEV- A3 | 4 | 4 | 4 (Iteration 1) | 5 | 5 |
| SUDEV- A4 | 3 | 5 (Iteration 1) | 4 | 5 | 5 |

**Table A11**
Completed linguistic evaluations of decision makers for the criteria

| DM1 | | | | |
|---|---|---|---|---|
| | DECARB | REENEX | ENEFF | SUDEV |
| DECARB | | H | VH | M |
| REENEX | H | | M | VH |
| ENEFF | VH | VH | | VH |
| SUDEV | S | M | S | |
| **DM3** | | | | |
| DECARB | | VH | VH | H |
| REENEX | M | | S | M |
| ENEFF | S | H | | VH |
| SUDEV | H | H | VH | |
| **DM4** | | | | |
| DECARB | | H | M | VH |
| REENEX | M | | H | H |
| ENEFF | VH | M | | H |
| SUDEV | H | H | H | |
| **DM5** | | | | |
| DECARB | | VH | M | H |
| REENEX | VH | | M | M |
| ENEFF | M | M | | VH |
| SUDEV | H | H | H | |
| **DM6** | | | | |
| DECARB | | M | H | H |
| REENEX | H | | H | VH |
| ENEFF | VH | VH | | H |
| SUDEV | M | M | S | |

**Table A12**
Quantum picture fuzzy numbers for the relation matrix

| DM1 | | | | |
|---|---|---|---|---|
| | DECARB | REENEX | ENEFF | SUDEV |
| DECARB | | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ |
| REENEX | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| ENEFF | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| SUDEV | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | |
| **DM3** | | | | |
| DECARB | | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| REENEX | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ |
| ENEFF | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| SUDEV | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | |
| **DM4** | | | | |
| DECARB | | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| REENEX | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| ENEFF | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| SUDEV | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | |
| **DM5** | | | | |
| DECARB | | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| REENEX | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ |
| ENEFF | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| SUDEV | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | |
| **DM6** | | | | |
| DECARB | | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| REENEX | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |

(*continued on next page*)

**Table A12** (*continued*)

| DM1 | | | | |
|---|---|---|---|---|
| | DECARB | REENEX | ENEFF | SUDEV |
| ENEFF | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| SUDEV | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | |

**Table A13**
Expert weighted quantum picture fuzzy rough sets for the direct relation matrix

| | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| DECARB | | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$ |
| REENEX | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | | $\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ |
| ENEFF | $\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.06,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.04,.10\rfloor}, \lceil\sqrt{.03},\sqrt{.09}\rfloor e^{j2\pi.\lceil.00,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.00,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ |
| SUDEV | $\lceil\sqrt{.02},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.01},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.00,.02\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$ | $\lceil\sqrt{.02},\sqrt{.10}\rfloor e^{j2\pi.\lceil.04,.20\rfloor}, \lceil\sqrt{.01},\sqrt{.06}\rfloor e^{j2\pi.\lceil.02,.12\rfloor}, \lceil\sqrt{.02},\sqrt{.15}\rfloor e^{j2\pi.\lceil.00,.05\rfloor}, \lceil\sqrt{.01},\sqrt{.09}\rfloor e^{j2\pi.\lceil.00,.03\rfloor}$ | |

**Table A14**
The defuzzified values of QPFRSs

| | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| DECARB | 0.000 | 0.068 | 0.066 | 0.060 |
| REENEX | 0.066 | 0.000 | 0.061 | 0.068 |
| ENEFF | 0.071 | 0.066 | 0.000 | 0.068 |
| SUDEV | 0.059 | 0.060 | 0.065 | 0.000 |

**Table A15**
The normalized relation matrix

| | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| DECARB | 0.000 | 0.351 | 0.341 | 0.308 |
| REENEX | 0.339 | 0.000 | 0.313 | 0.349 |
| ENEFF | 0.347 | 0.323 | 0.000 | 0.330 |
| SUDEV | 0.318 | 0.328 | 0.354 | 0.000 |

**Table A16**
Sj, kj, qj, and wj values for the relationship degrees of each criterion

| DECARB | Sj | kj | qj | Wj | REENEX | Sj | Kj | qj | Wj |
|---|---|---|---|---|---|---|---|---|---|
| REENEX | 0.351 | 1.000 | 1.000 | 0.432 | SUDEV | 0.349 | 1.000 | 1.000 | 0.432 |
| ENEFF | 0.341 | 1.341 | 0.746 | 0.322 | DECARB | 0.339 | 1.339 | 0.747 | 0.323 |
| SUDEV | 0.308 | 1.308 | 0.570 | 0.246 | ENEFF | 0.313 | 1.313 | 0.569 | 0.246 |
| **ENEFF** | **Sj** | **kj** | **qj** | **wj** | **SUDEV** | **Sj** | **Kj** | **qj** | **Wj** |
| DECARB | 0.347 | 1.000 | 1.000 | 0.431 | ENEFF | 0.354 | 1.000 | 1.000 | 0.430 |
| SUDEV | 0.330 | 1.330 | 0.752 | 0.324 | REENEX | 0.328 | 1.328 | 0.753 | 0.324 |
| REENEX | 0.323 | 1.323 | 0.568 | 0.245 | DECARB | 0.318 | 1.318 | 0.571 | 0.246 |

**Table A17**
Relation Matrix with the values of wj and the impact directions

| | DECARB | REENEX | ENEFF | SUDEV | Impact directions |
|---|---|---|---|---|---|
| DECARB | | 0.432 | 0.322 | 0.246 | C1→C2 |
| REENEX | 0.323 | | 0.246 | 0.432 | C2→C4 |
| ENEFF | 0.431 | 0.245 | | 0.324 | C3→C1 |
| SUDEV | 0.246 | 0.324 | 0.430 | | C4→C3 |

**Table A18**
Quantum picture fuzzy numbers for the decision matrix

| | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| DM2 | | | | |
| A1 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| A2 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| A3 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A4 | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ |
| **DM3** | | | | |
| A1 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A2 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A3 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A4 | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| **DM4** | | | | |
| A1 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A2 | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A3 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.20}e^{j2\pi..45},\\ \sqrt{.13}e^{j2\pi..28},\\ \sqrt{.42}e^{j2\pi..17},\\ \sqrt{.25}e^{j2\pi..10}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A4 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| **DM5** | | | | |
| A1 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| A2 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.25}e^{j2\pi..50},\\ \sqrt{.15}e^{j2\pi..31},\\ \sqrt{.37}e^{j2\pi..12},\\ \sqrt{.23}e^{j2\pi..07}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |

(*continued on next page*)

**Table A18** (*continued*)

| DM2 | | | | |
|---|---|---|---|---|
| | DECARB | REENEX | ENEFF | SUDEV |
| A3 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| A4 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| **DM6** | | | | |
| A1 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ |
| A2 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| A3 | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |
| A4 | $\begin{bmatrix}\sqrt{.30}e^{j2\pi..55},\\ \sqrt{.19}e^{j2\pi..34},\\ \sqrt{.32}e^{j2\pi..07},\\ \sqrt{.19}e^{j2\pi..04}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ | $\begin{bmatrix}\sqrt{.36}e^{j2\pi..6},\\ \sqrt{.22}e^{j2\pi..37},\\ \sqrt{.26}e^{j2\pi..02},\\ \sqrt{.16}e^{j2\pi..01}\end{bmatrix}$ |

**Table A19**
Expert weighted quantum picture fuzzy rough sets for the decision matrix

| | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| A1 | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.04},\sqrt{.08}\rfloor e^{j2\pi.\lceil.06,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.04,.09\rfloor}, \lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.00,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.00,.02\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ |
| A2 | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$ | $\lceil\sqrt{.02},\sqrt{.10}\rfloor e^{j2\pi.\lceil.04,.20\rfloor}, \lceil\sqrt{.01},\sqrt{.06}\rfloor e^{j2\pi.\lceil.02,.12\rfloor}, \lceil\sqrt{.02},\sqrt{.15}\rfloor e^{j2\pi.\lceil.00,.05\rfloor}, \lceil\sqrt{.01},\sqrt{.09}\rfloor e^{j2\pi.\lceil.00,.03\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ |
| A3 | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$ | $\lceil\sqrt{.02},\sqrt{.10}\rfloor e^{j2\pi.\lceil.04,.20\rfloor}, \lceil\sqrt{.01},\sqrt{.06}\rfloor e^{j2\pi.\lceil.02,.12\rfloor}, \lceil\sqrt{.02},\sqrt{.15}\rfloor e^{j2\pi.\lceil.00,.05\rfloor}, \lceil\sqrt{.01},\sqrt{.09}\rfloor e^{j2\pi.\lceil.00,.03\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ |
| A4 | $\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor}, \lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ | $\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor}, \lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor}, \lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}, \lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$ |

**Table A20**
The defuzzified decision values

| Criteria/Alternatives | DECARB | REENEX | ENEFF | SUDEV |
|---|---|---|---|---|
| A1 | 0.068 | 0.064 | 0.066 | 0.068 |
| A2 | 0.066 | 0.062 | 0.063 | 0.068 |
| A3 | 0.067 | 0.061 | 0.063 | 0.068 |
| A4 | 0.060 | 0.068 | 0.066 | 0.065 |

**Table A21**
Si,Ri, and Qi values

| | Si | Ri | Qi (v:.1) | Qi (v:.2) | Qi (v:.3) | Qi (v:.4) |
|---|---|---|---|---|---|---|
| A1 | 0.124 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 |
| A2 | 0.514 | 0.250 | 0.995 | 0.993 | 0.991 | 0.989 |
| A3 | 0.523 | 0.251 | 1.000 | 1.000 | 1.000 | 1.000 |
| A4 | 0.499 | 0.250 | 0.988 | 0.983 | 0.977 | 0.972 |
| | **Qi (v:.5)** | **Qi (v:.6)** | **Qi (v:.7)** | **Qi (v:.8)** | **Qi (v:.9)** | **Qi (v:1)** |
| A1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| A2 | 0.987 | 0.985 | 0.983 | 0.981 | 0.979 | 0.977 |
| A3 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| A4 | 0.967 | 0.961 | 0.956 | 0.950 | 0.945 | 0.939 |

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