RESEARCH ARTICLE

Enhancing physical layer security in AF relay–assisted multicarrier wireless transmission

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Abstract

In this paper, we study the physical layer security problem in the dual-hop orthogonal frequency division multiplexing–based wireless communication system. First, we consider a single-user single-relay system and study a joint power optimization problem at the source and relay subject to individual power constraint at the 2 nodes. The aim is to maximize the end-to-end secrecy rate with optimal power allocation over different subcarriers. Later, we consider a more general multiuser multirelay scenario. Under high signal-to-noise ratio approximation for end-to-end secrecy rate, an optimization problem is formulated to jointly optimize power allocation at the base station, the relay selection, subcarrier assignment to users, and the power loading at each of the relaying node. The target is to maximize the overall security of the system subject to independent power budget limits at each transmitting node and the orthogonal frequency division multiple access–based exclusive subcarrier allocation constraints. A joint optimization solution is obtained through the duality theory. Dual decomposition allows to exploit convex optimization techniques to find the power loading at the source and relay nodes. Furthermore, an optimization for power loading at relaying nodes along with relay selection and subcarrier assignment for the fixed power allocation at the base station is also studied. A suboptimal scheme that explores joint power allocation at all transmitting nodes for the fixed subcarrier allocation and relay assignment is investigated. Finally, simulation results are presented to validate the performance of the proposed schemes.

1 | INTRODUCTION

Dual-hop communication has recently gained significant attention in the field of wireless communication due to its better performance over single-hop communication.[1] In dual-hop communication, a relay is used as an intermediate node between a sender and a receiver. It is generally used to enhance throughput, reduce power consumption, and to increase coverage area at the cell edges. There are 2 types of relaying protocols that are widely used: amplify and forward (AF) and decode and forward (DF). The AF relaying protocol first receives signal from the source and then forwards it to the destination with amplification, whereas DF relaying protocol first receives signal from the source, decodes it, reencodes it, and then forwards the resultant signal to the destination.[2]

The broadcast nature of wireless communication provides many exciting opportunities; however, it makes the security of link a challenging issue. Wireless communications can potentially be attacked by malicious nodes, and therefore, security issues have taken an important role in today's communications.[3] A promising technique for achieving secure

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communications is physical layer security (PLS).[4] A wireless link is considered to be secure if it provides a positive nonzero secrecy rate, and a link with higher secrecy rate is known to be a more secure link.[5]

To provide PLS in dual-hop single-carrier networks, resource allocation has been widely studied under DF relaying protocol.[6-12] Mo et al[6] and Deng et al[7] studied the problem of optimal relay placement to enhance PLS. Wang et al[8] considered the joint relay selection and power optimization to maximize the system's secrecy rate. Furthermore, Guo et al[9] proposed a joint relay and jammers selection with power optimization. Recently, Yang et al[12] discussed the relay selection in the presence of adaptive eavesdropper. The dual-hop transmission under AF protocols has become much attractive due to its simple implementation.[10] However, the resource allocation in AF relay (AR)–enhanced networks has always been a challenging task.[11] Different aspects of PLS in AF-based single-carrier systems have been studied in other works.[13-17] Huang et al[13] investigated the impact of using an untrusted AR on secure communication and derived the exact secrecy outage probability under different transmission scenarios. With multiple trusted relays, Fan et al[14] proposed the different relay selection strategies to enhance the PLS in multiuser cooperative relay networks. Akhtar et al[15] focused on the achievability of secrecy rate under different channel conditions. The suboptimal relay selection with fairness is studied in the work of Du et al.[16] The relay transmit power optimization protocols for secrecy maximization under both AF and DF have been studied in the work of Dong et al.[17]

The multicarrier transmission has become a fundamental choice for the next-generation wireless communication networks because of its ability to combat multipath fading effects, high spectral efficiency, and provision of flexibility in resource allocation.[18] To provide PLS in multicarrier systems, resource optimization is one of the popular techniques and has been studied in other works.[19-25] In the work of Li et al,[19] a dynamic subcarrier allocation for secure transmission is studied in the presence of passive eavesdropper. The proposed scheme utilizes the channel state information between legitimate users and drops out highly faded subcarrier and modifies the modulation scheme for the remaining good subcarriers to achieve a better secrecy rate. Furthermore, Bai et al[20] provided an optimal subcarrier allocation for outage probability minimization with secrecy constraint. Benfarah et al,[21] proposed the optimal power allocation with subcarrier allocation to maximize the sum rate with outage probability and fairness constraints. In the work of Wang et al,[22] 2 categories of users were considered: the secure users and the nonsecure users. The task was to maximize the throughput of nonsecure users via optimal power allocation subject to guaranteed average secrecy rate to secure users. Abedi et al[23] extended the previous work to maximize the secrecy rate in the presence of active eavesdropper that has the capability to jam the secret user transmission. In the presence of both the active and passive eavesdroppers, the power allocation over subcarriers to maximize the average secrecy rate has been investigated in the work of Poor et al.[24] Furthermore, Ng et al[25] considered multiple eavesdroppers and optimized subcarrier assignment, power allocation, and secrecy data rate to maximize the energy efficiency.

1.1 | Related work and contributions

Under the umbrella of orthogonal frequency division multiple access, resource allocation for PLS in a dual hop with DF protocol has been studied in other works.[26-28] The power allocation problem to maximize the secrecy rate was investigated in the work of Jeong and Kim.[26] The extension to this work with joint subcarrier allocation and power loading was made in the work of Wang et al.[27] Recently, Saini et al[28] proposed the 2 different power optimization schemes at source and relay under individual power constraint: one achieves sum secrecy rate maximization in the presence of untrusted users, whereas the other achieves fairness for the minimum requirement of secrecy per user. The dual-hop communication under DF relaying is allowed for the trusted relaying nodes only. However, if the relay node is untrusted, AF protocol becomes the better choice as it does not require any decoding at the relay. However, the resource allocation schemes designed to enhance PLS under DF transmission cannot be directly applied to the AF scenario.

Recently, different works on PLS in dual-hop systems under the AF relaying protocol have been reported.[29-32] The resource optimization in orthogonal frequency division multiplexing (OFDM)–based single-user single-relay systems was considered in the work of Jindal and Bose.[29] The authors studied the subcarrier assignment and power allocation strategies under a total system power constraint. The optimization under sum power constraint provides a good analysis of power allocation; however, it may not be an attractive solution for practical systems. Furthermore, Cai et al[30] investigated the power allocation at source node and subcarriers allocation among users in a single-relay multiple-user system. Recently, Aman et al[31] extended the work to a multirelay scenario and considered the relay assignment and power allocation problem. However, both the works of Cai et al[30] and Aman et al[31] considered power allocation at the source node only while the power optimization at the relaying node(s) was missing. The power optimization only at the source node

simplifies the solution at the cost of degradation in performance. More recently, Jindal and Bose[32] investigated the power allocation at the source and relay nodes under a single-user single-relay scenario. The authors proposed a suboptimal solution through an alternate optimization approach. A joint optimization of power allocation at the source and relay nodes along with subcarrier assignment and relay selection can provide much more benefits. This joint optimization is a challenging task and, to the best of authors' knowledge, has not been investigated yet.

In this work, our aim is to maximize the sum secrecy rate under AF relaying protocol in a single cell downlink transmission. We first consider the joint power allocation at the source and relay nodes subject to separate power constraint at each node. The end-to-end secrecy rate under AF protocol depends on both hops, ie, the power allocation at the 2 nodes is coupled with each other. Thus, instead of separate power optimization[32] at the source and the relay, we propose a joint optimization solution. Then, we consider a joint subcarrier allocation, relay selection, and power allocation problem in a multiuser multirelay system. Various solution schemes are proposed to efficiently solve the problem. Our contributions are summarized as follows.

- We solve a joint power allocation problem in an OFDM-based dual-hop network to optimize the power distribution among different subcarriers at the source node and the relay node. An efficient solution is obtained through Karush-Kuhn-Tucker (KKT) optimality conditions.
- Later, a novel joint optimization problem is formulated, which considers the power allocation at the source node, the optimal relay assignment to users, the subcarrier allocation to each assigned relaying node, the power allocation at each relay node, and the subcarrier allocation to each user subject to separate power constraint at the source and each of the relaying node.
- A joint solution of the mixed integer programming problem is obtained through efficient dual decomposition techniques to maximize the overall system's secrecy rate.
- To look into the effect of power optimization at the relaying nodes only, we redefine the joint problem for uniform power allocation at the source node, and similar techniques are used to solve this problem.
- Finally, a low complexity suboptimal algorithm is proposed, which optimizes the power at the source and the multiple relaying nodes for the predefined subcarrier allocation and the relay assignment.
- Extensive simulation results are presented to evaluate the performance of the proposed schemes.

The remainder of this paper is organized as follows. The joint power allocation at the source node and the relay node in a single-user single-relay case is presented in Section 2. Under a multiuser multirelay system, the proposed framework is elaborated in Section 3. Section 4 includes the proposed solution for power allocation at the relaying nodes along with subcarrier allocation, and the relay assignment where the problem of power allocation at the source and the multiple relaying nodes without optimizing other parameters is considered in Section 5. Finally, the simulation results and the conclusion are presented in Sections 6 and 7, respectively.

2 | JOINT POWER ALLOCATION AT THE RELAY AND THE SOURCE NODE

2.1 | System model and problem formulation

In this section, we consider a dual-hop multicarrier system that consists of a source node (S), an AF relay node (AR), a destination node (D), and an eavesdropper (Eve), as shown in Figure 1. We assume that all devices are equipped with single antenna and D and Eve are colocated such that the direct path from S to D and S to Eve is missing due to large distance.[27,29,30] The channel gains of the ith subcarrier over S-to-AR, AR-to-D, and AR-to-Eve links are denoted by h_i, g_i,
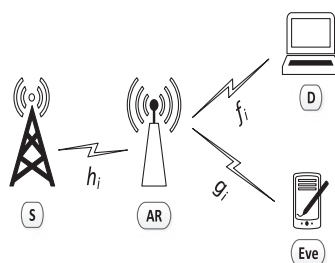

FIGURE 1 Orthogonal frequency division multiplexing–based single-user single-relay system. AR, amplify-and-forward relay

and f_i, respectively. In the first transmission slot, the AR receives a message signal over the ith subcarrier and retransmits with amplification factor Q_i given by

$$Q_i = \sqrt{\frac{q_i}{p_i|h_i|^2 + \sigma^2}}, \tag{1}$$

where p_i and q_i are the power loading over the ith carrier at S and AR, respectively, and σ^2 denotes the variance of additive white Gaussian noise.

The received signal-to-noise ratio (SNR) at the D over the ith subcarrier can be expressed as

$$\text{SNR}_i^D = \frac{Q_i^2 p_i |h_i|^2 |g_i|^2}{Q_i^2 |g_i|^2 \sigma^2 + \sigma^2}. \tag{2}$$

Similarly, the received SNR at the Eve is given as

$$\text{SNR}_i^E = \frac{Q_i^2 p_i |h_i|^2 |f_i|^2}{Q_i^2 |f_i|^2 \sigma^2 + \sigma^2}. \tag{3}$$

The secrecy rate over the ith subcarrier can be expressed as

$$SR_i = \log_2\left(1 + \text{SNR}_i^D\right) - \log_2\left(1 + \text{SNR}_i^E\right). \tag{4}$$

Letting N be the total number of subcarriers, the sum secrecy rate under high SNR approximation can be written as[14]

$$SR_{\text{sum}} = \frac{1}{2}\sum_{i=1}^{N}\underbrace{\log_2\left(\frac{G_i + H_iG_ip_i + G_iF_iq_i}{F_i + H_iF_ip_i + G_iF_iq_i}\right)}_{SR_i}, \tag{5}$$

where $H_i = \frac{|h_i|^2}{\sigma^2}$, $G_i = \frac{|g_i|^2}{\sigma^2}$, $F_i = \frac{|f_i|^2}{\sigma^2}$, and the term $\frac{1}{2}$ appears due to half-duplex relay transmission.

Our target is to maximize the sum secrecy rate of the system by optimizing power over the subcarriers at the S and at the AR under individual power constraints. Thus, the optimization problem becomes

$$\max_{p_i, q_i} \quad \sum_{i=1}^{N} SR_i$$
$$\sum_{i=1}^{N} p_i \leq P_t, \quad \sum_{i=1}^{N} q_i \leq Q_t. \tag{6}$$

The first constraint ensures that the total power allocated to all the subcarriers at S must be within the total available power P_t. Similarly, the second constraint ensures that the allocated power over all subcarriers at the AR node should not exceed the maximum limit Q_t.

2.2 | Proposed optimization scheme

The problem (6) is a convex optimization problem, and we use the duality theory to obtain the solution. The optimal power loading can be obtained from the following dual problem:

$$\min_{\lambda \geq 0, V \geq 0} \quad \max_{p_i \geq 0, q_i \geq 0} \sum_{i=1}^{N} SR_i + \lambda\left(P_t - \sum_{i=1}^{N} p_i\right)$$
$$+ V\left(Q_t - \sum_{i=1}^{N} q_i\right), \tag{7}$$

where λ and V are the associated dual variables. Removing the constant terms, the problem can be rewritten as

$$\min_{\lambda \geq 0, V \geq 0} \quad \sum_{i=1}^{N} \max_{p_i \geq 0, q_i \geq 0} \log_2\left(\frac{G_i + H_iG_ip_i + G_iF_iq_i}{F_i + H_iF_ip_i + G_iF_iq_i}\right)$$
$$- \lambda p_i - Vq_i. \tag{8}$$

TABLE 1

A_i	$H_i^2 \lambda (H_i G_i^3 V - H_i G_i F_i^2 V - H_i G_i^2 F_i V + H_i F_i^3 V$ $+ G_i^3 F_i \lambda + G_i F_i^3 \lambda - 2G_i^2 F_i^2 \lambda)$
B_i	$(2H_i^3 G_i^2 + 2H_i^3 F_i^2 - 4H_i^3 G_i F_i - 2H_i^2 G_i^2 F_i \lambda V + 3H_i^2 G_i^3 \lambda V - 3H_i^2 G_i F_i^2 \lambda V + H_i^2 F_i^3 \lambda V$ $+ 2H_i G_i^3 F_i \lambda^2 + 2H_i G_i F_i^2 \lambda^2 - 4H_i G_i^2 F_i^2 \lambda^2 - H_i^3 G_i^3 V - 3H_i^3 G_i F_i^2 V + 3H_i^3 G - i^2 F_i V + H_i F_i^3 V)$
C_i	$(H_i^2 G_i^2 + H_i^2 F_i^2 - 2H_i^2 G_i F_i + H_i G_i^3 \lambda V - 2H_i G_i^2 F_i \lambda V + H_i G_i F_i \lambda V - 2H_i G_i F_i^2 \lambda V + G_i^3 F_i \lambda^2$ $+ G_i F_i^3 \lambda^2 - 2G_i^2 F_i^2 \lambda^2 - H_i^2 G_i^3 V - 3H_i^2 G_i F_i^2 V + 3H_i^2 G_i^2 F_i V + H_i^2 F_i^3 V)$

Applying KKT conditions to the internal maximization, we obtain

$$p_i^* = \left(\frac{-B_i + \sqrt{B_i^2 - 4A_i C_i}}{2A_i} \right)^+,$$

(9)

and

$$q_i^* = \left(\frac{\lambda D_i}{V E_i} + \frac{\lambda B_i + \sqrt{(\lambda B_i)^2 - 4A_i C_i \lambda^2}}{2V A_i} \right)^+,$$

(10)

where $(x)^+ = \max(0, x)$, $D_i = (F_i - G_i)$, $E_i = (G_i H_i - H_i F_i)$ and the values of A_i, B_i, and C_i are given in Table 1.

The problem in (8) becomes

$$\min_{\lambda \geq 0, V \geq 0} \quad \sum_{i=1}^{N} \log_2 \left(\frac{G_i + H_i G_i p_i^* + G_i F_i q_i^*}{F_i + H_i F_i p_i^* + G_i F_i q_i^*} \right) - \lambda p_i^* - V q_i^*.$$

To find the dual variables, we use the following iterative subgradient updates[33-35]:

$$\lambda(m) = \lambda(m-1) + \delta \left(P_t - \sum_{i=1}^{N} p_i^* \right),$$

(11)

$$V(m) = V(m-1) + \delta \left(Q_t - \sum_{i=1}^{N} q_i^* \right),$$

(12)

where m represents the mth iteration and δ is the step size. In each update of dual variables, the optimum power allocation at the base station (BS) and the relay are obtained from (9) and (10). At convergence, the optimum values of dual variables and of power variables are obtained.

3 | JOINT SUBCARRIERS ALLOCATION, RELAY SELECTION, AND POWER ALLOCATION

In this section, we consider multiuser, multirelay, and multicarrier dual-hop communication with a single BS, K as the number of secret users, J as the number of ARs, N as the number of subcarriers, and a single Eve, as shown in Figure 2. The channel gain from the BS to the jth AR node on the ith subcarrier is denoted by $h_{i,j}$. The channel gain from the jth AR node to the kth user on the ith subcarrier is denoted by $g_{i,j,k}$, the corresponding channel gain from the jth AR node to Eve is denoted by $f_{i,j}$, and $u_{i,j}$ is the power allocated over the ith subcarrier at the jth relay. With this, the secrecy rate over ith subcarrier at the kth user communicated through jth relay can be expressed as

$$SR_{i,j,k} = \frac{1}{2} \log_2 \left(\frac{b_{i,j,k} + a_{i,j} b_{i,j,k} p_i + u_{i,j} b_{i,j,k} c_{i,j}}{c_{i,j} + a_{i,j} c_{i,j} p_i + u_{i,j} b_{i,j,k} c_{i,j}} \right),$$

(13)

where $a_{i,j} = \frac{|h_{i,j}|^2}{\sigma^2}$, $b_{i,j} = \frac{|g_{i,j,k}|^2}{\sigma^2}$, and $c_{i,j} = \frac{|f_{i,j}|^2}{\sigma^2}$. We adopt a fully flexible AR allocation strategy where a relaying node can be allocated to more than one user, and each user can be served with multiple AR nodes over different subcarriers. Furthermore, a subcarrier is allocated to the same user over the 2 hops of transmission.* On account of subcarrier allocation and AR selection, we define 2 binary variables: $\alpha_{i,k} \in [0, 1]$ such that $\alpha_{i,k} = 1$ when the ith subcarrier is allocated to

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*The information received over the ith subcarrier at the first hop can be forwarded over a different carrier in the second hop; however, it is beyond the scope of this work.
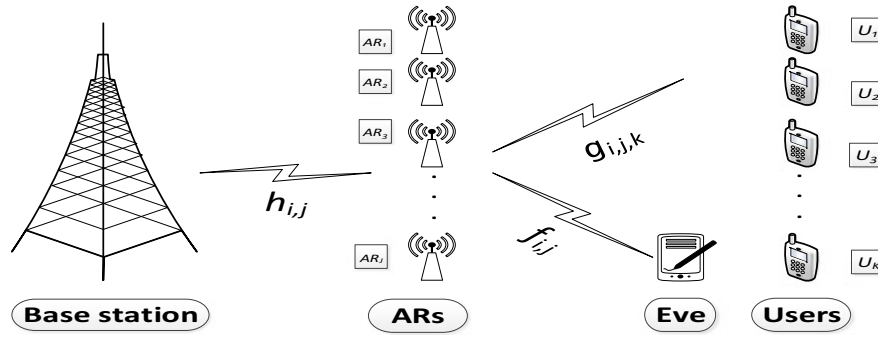
FIGURE 2 Orthogonal frequency division multiple access–based multiuser multirelay system. AR, amplify-and-forward relay

the kth user and zero otherwise and $\beta_{j,k} \in [0, 1]$ such that $\beta_{j,k} = 1$ when the jth AR is allocated to the kth user and zero otherwise. With this, the sum secrecy rate of the system can be expressed as

$$SR_{sum} = \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} SR_{i,j,k}. \tag{14}$$

3.1 | Problem formulation

The aim is to maximize SR_{sum} with jointly optimizing the AR assignment, subcarrier allocation, BS's transmit power loading, and power allocation at the relaying nodes over different subcarriers. Let P_t and $Q_{t,j}$ be the total power available at the BS and the jth AR, respectively. Then, the joint subcarrier allocation, AR assignment, and power loading optimization can be formulated as

$$\max_{(p_i, u_{i,j}, \alpha_{i,k}, \beta_{j,k})} \quad SR_{sum}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \alpha_{i,k} = 1, \quad \forall i,$$

$$\sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} p_i \le P_t, \tag{15}$$

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} u_{i,j} \le Q_{t,j} \quad \forall j.$$

The first constraint ensures that a particular subcarrier cannot be assigned to more than one user. The second constraint represents that the sum transmit power on all subcarriers at the BS should be less than or equal to a maximum power limit P_t and the last constraint guarantees that the total transmit power over different subcarriers at the jth AR should be less than or equal to a maximum power budget $Q_{t,j}$.

3.2 | Proposed solution

The problem (15) is a mixed binary integer programming problem, and a vast search over all variables is needed to find an optimal solution. Due to Yu et al,[34] the difference between the solution of dual problem and the solution of primal problem[†] becomes zero when we have sufficiently large number of subcarriers in OFDM-based transmission regardless of the convexity of original problem. The dual problem associated with the primal problem (15) can be defined as

$$\min_{(\lambda \ge 0, \ V_j \ge 0)} \quad D(\lambda, V_j), \tag{16}$$

[†]Commonly known as duality gap.

where λ and V_j are the dual variables, and the dual function $D(\lambda, V_j)$ can be expressed as

$$D(\lambda, V_j) = \max_{(p_i, u_{i,j}, \alpha_{i,k}, \beta_{j,k})} L\left(p_i, u_{i,j}, \alpha_{i,k}, \beta_{j,k}\right)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \alpha_{i,k} = 1, \quad \forall i, \tag{17}$$

with

$$L(p_i, u_{i,j}, \alpha_{i,k}, \beta_{j,k}) = \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K} \alpha_{i,k}\beta_{j,k} SR_{i,j,k} + \lambda\left(P_t - \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K} \alpha_{i,k}\beta_{j,k}p_i\right)$$

$$+ \sum_{j=1}^{J} V_j\left(Q_{t,j} - \sum_{i=1}^{N}\sum_{k=1}^{K} \alpha_{i,k}\beta_{j,k}u_{i,j}\right). \tag{18}$$

To solve the dual problem, we first solve the dual function $D(\lambda, V_j)$, and similar to the work of Sidhu et al,[35] we adopt the dual decomposition approach. The problem in (17) can be rewritten as

$$D(\lambda, V_j) = \max_{(p_i, u_{i,j}, \alpha_{i,k}, \beta_{j,k})} \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K} \alpha_{i,k}\beta_{j,k}(SR_{i,j,k}, -\lambda p_i - V_j u_{i,j}) + \lambda P_t + V_j Q_{t,j}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \alpha_{i,k} = 1, \quad \forall i. \tag{19}$$

Now, for any given subcarrier allocation and relay assignment, the optimal power allocation at the BS and the jth AR can be obtained from

$$\max_{p_i \geq 0, u_{i,j} \geq 0} \left(\log_2\left(\frac{b_{i,j,k} + a_{i,j}b_{i,j,k}p_i + u_{i,j}b_{i,j,k}c_{i,j}}{c_{i,j} + a_{i,j}c_{i,j}p_i + u_{i,j}b_{i,j,k}c_{i,j}}\right) - \lambda p_i - V_j u_{i,j}\right). \tag{20}$$

The problem (20) is convex, and the closed-form solution can be obtained by exploiting the standard techniques similar to Section 2.2. Applying KKT conditions, we get

$$p_i^* = \left(\frac{-Y_i + \sqrt{Y_i^2 - 4X_i Z_i}}{2X_i}\right)^+, \tag{21}$$

where the values of X_i, Y_i, and Z_i are given in Table 2, and

$$u_{i,j}^* = \left(\frac{\lambda}{V_j}\left(\frac{U_i}{W_i} + \frac{Y_i + \sqrt{Y_i^2 - 4X_i Z_i}}{2X_i}\right)\right)^+, \tag{22}$$

with $U_i = (c_{i,j} - b_{i,j,k})$ and $W_i = (b_{i,j,k}a_{i,j,k} - a_{i,j,k}c_{i,j,k})$. Putting p_i^* and $u_{i,j}^*$ into (19), the dual function can be rewritten as

$$D(\lambda, V_j) = \max_{(\alpha_{i,k}, \beta_{j,k})} \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K} \alpha_{i,k}\beta_{j,k}\left(SR_{i,j,k}^* - \lambda p_i^* - V_j u_{i,j}^*\right)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \alpha_{i,k} = 1, \quad \forall i, \tag{23}$$

where $SR_{i,j,k}^*$ is given by

$$SR_{i,j,k}^* = \log_2\left(\frac{b_{i,j,k} + a_{i,j}b_{i,j,k}p_i^* + u_{i,j}^*b_{i,j,k}c_{i,j}}{c_{i,j} + a_{i,j}c_{i,j}p_i^* + u_{i,j}^*b_{i,j,k}c_{i,j}}\right) - \lambda p_i^* - V_j u_{i,j}^*.$$

TABLE 2

X_i	$(a_{i,j}^3 b_{i,j,k}^3 \lambda V_j - a_{i,j}^3 b_{i,j,k} c_{i,j}^2 \lambda V_j - a_{i,j}^3 b_{i,j,k}^2 c_{i,j} \lambda V_j$
	$+ a_{i,j}^3 c_{i,j}^3 \lambda V_j + a_{i,j}^2 b_{i,j,k}^3 c_{i,j} \lambda^2 + a_{i,j}^2 b_{i,j,k} c_{i,j}^3 \lambda^2 - 2a_{i,j}^2 b_{i,j,k}^2 c_{i,j}^2 \lambda^2)$
Y_i	$(2a_{i,j}^3 b_{i,j,k}^2 + 2a_{i,j}^3 c_{i,j}^2 - 4a_{i,j}^3 b_{i,j,k} c_{i,j} - 2a_{i,j}^2 b_{i,j,k}^2 c_{i,j} \lambda V_j + 3a_{i,j}^2 b_{i,j,k}^3 \lambda V_j - 3a_{i,j}^2 b_{i,j,k} c_{i,j}^2 \lambda V_j + a_{i,j}^2 c_{i,j}^3 \lambda V_j$
	$+ 2a_{i,j} b_{i,j,k}^3 c_{i,j} \lambda^2 + 2a_{i,j} b_{i,j,k} c_{i,j}^3 \lambda^2 - 4a_{i,j} b_{i,j,k}^2 c_{i,j}^2 \lambda^2 - a_{i,j}^3 b_{i,j,k}^3 V_j - 3a_{i,j}^3 b_{i,j,k} c_{i,j}^2 V_j + 3a_{i,j,k}^3 b_{i,j,k}^2 c_{i,j} V_j + a_{i,j,k} c_{i,j}^3 V_j)$
Z_i	$(a_{i,j}^2 b_{i,j,k}^2 + a_{i,j}^2 c_{i,j}^2 - 2a_{i,j}^2 b_{i,j,k} c_{i,j} + a_{i,j} b_{i,j,k}^3 \lambda V_j - 2a_{i,j} b_{i,j,k}^2 c_{i,j} \lambda V_j + a_{i,j} b_{i,j,k} c_{i,j}^2 \lambda V_j - 2a_{i,j} b_{i,j,k} c_{i,j}^2 \lambda V_j$
	$+ b_{i,j,k}^3 c_{i,j} \lambda^2 + b_{i,j,k} c_{i,j}^3 \lambda^2 - 2b_{i,j,k}^2 c_{i,j}^2 \lambda^2 - a_{i,j}^2 b_{i,j,k}^3 V_j - 3a_{i,j}^2 b_{i,j,k} c_{i,j}^2 V_j + 3a_{i,j}^2 b_{i,j,k}^2 c_{i,j} V_j + a_{i,j}^2 c_{i,j}^3 V_j)$

Now, we need to find the optimal subcarrier allocation and relay assignment. For immediate recovery of the binary variables $\alpha_{i,k}$ and $\beta_{j,k}$, we define a new variable $\pi_{i,j,k} \in \{0,1\}$ such that $\pi_{i,j,k} = 1$ if $\alpha_{i,k}\beta_{j,k} = 1$ and zero otherwise. The above problem can be rewritten as

$$D(\lambda, V_j) = \max_{\pi_{i,j,k}} \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \pi_{i,j,k} \left(SR_{i,j,k}^* - \lambda p_i^* - V_j u_{i,j}^* \right)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \sum_{j=1}^{J} \pi_{i,j,k} = 1, \quad \forall i. \tag{24}$$

The constraint in the above optimization ensures that each subcarrier is assigned to one relay and one user. The optimum solution of the above problem is to assign a subcarrier AR pair (i, j) to user k, which maximizes $SR_{i,j,k}^*$, ie,

$$(i^*, j^*, k^*) = \arg\max_{i,j} \quad SR_{i,j,k}^*. \tag{25}$$

Thus, we have

$$\pi_{i,j,k}^* = \begin{cases} 1, & \text{for} (i, j, k) = (i^*, j^*, k^*) \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

Now, the optimum subcarrier allocation and relay assignment are obtained. Let $\alpha_{i,k}^*$ and $\beta_{j,k}^*$ denote the optimal assignment variables. Thus, substituting $p_i *$, $u_{i,j}^*$, $\alpha_{i,k}^*$, and $\beta_{j,k}^*$ into (18), we obtain the dual function.

Next, similar to (11) and (12), we solve the dual problem (16) with the subgradient method.[33-35] The subgradient updates at $(m+1)$th iteration are

$$\lambda(m+1) = \lambda(m) + \delta \left(P_t - \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} p_i \right),$$

$$V_j(m+1) = V_j(m) + \delta \left(Q_{t,j} - \sum_{i=1}^{N} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} u_{i,j} \right), \forall j.$$

In each subgradient update, the values of power variables as well as relay selection and subcarrier assignment are obtained from (20), (21), and (25). The program is terminated at the convergence, and the proposed joint optimization algorithm is completed.

4 | OPTIMIZATION AT THE RELAY NODES FOR FIXED POWER ALLOCATION AT BS

Cai et al[30] and Aman et al[31] considered the power optimization at the BS for uniform distribution at the relay. The dynamic relay selection and subcarrier allocation strategy adopted in this work may assign a relay to multiple users and each relay may have different number of subcarriers. Thus, the power optimization at each relaying node with independent

power constraint becomes more important. In this section, we consider the joint optimization over power allocation at the relaying node, the relay selection, and the subcarrier assignment for the uniform power allocation at the BS, ie, $p_i = PT/N, \forall i$. The corresponding optimization problem can be written as

$$
\begin{aligned}
\max_{(u_{i,j}, \alpha_{i,k}, \beta_{j,k})} \quad & \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} \\
& \log_2 \left(\frac{b_{i,j,k} + a_{i,j} b_{i,j,k} p_i + u_{i,j} b_{i,j,k} c_{i,j}}{c_{i,j} + a_{i,j} c_{i,j} p_i + u_{i,j} b_{i,j,k} c_{i,j}} \right) \\
\text{s.t.} \quad & \sum_{k=1}^{K} \alpha_{i,k} = 1, \quad \forall i, \\
& \sum_{i=1}^{N} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} u_{i,j} \leq Q_{t,j} \quad \forall j.
\end{aligned}
\tag{27}
$$

This is a binary integer programming problem. Similar to Section 3.2, we adopt the dual decomposition approach. For any given relay assignment and subcarrier allocation, the power optimization at different relays can be obtained by solving following J subproblems:

$$
\max_{u_{i,j} \geq 0} \left(\log_2 \left(\frac{b_{i,j,k} + a_{i,j} b_{i,j,k} p_i + u_{i,j} b_{i,j,k} c_{i,j}}{c_{i,j} + a_{i,j} c_{i,j} p_i + u_{i,j} b_{i,j,k} c_{i,j}} \right) - \zeta_j u_{i,j} \right),
$$

$\forall j \in \{1, 2, \dots, J\}$ and ζ_j is the Lagrange multiplier corresponding to jth relay power constraint. The resultant value of $u_{i,j}^*$ is given as

$$
u_{i,j}^* = \left(\frac{-2 a_{i,j} b_{i,j,k} c_{i,j}^3 p_i + \sqrt{\left(2 a_{i,j} b_{i,j,k} c_{i,j}^3 p_i \right)^2 - 4 b_{i,j,k}^2 c_{i,j}^2 \Omega_i}}{2 b_{i,j,k}^2 c_{i,j}^2} \right)^+,
\tag{28}
$$

where $\Omega_i = c_{i,j}^2 + (a_{i,j} c_{i,j} p_i)^2 + 2 a_{i,j} c_{i,j}^2 p_i - \frac{1}{\zeta_j} \left(b_{i,j,k} c_{i,j}^2 - a_{i,j} b_{i,j,k} c_{i,j}^2 p_i + b_{i,j,k}^2 c_{i,j} + a_{i,j} b_{i,j,k}^2 c_{i,j} p_i \right)$. Now, similar to the previous section, we substitute the value of power variable in the corresponding dual function and the optimal relay assignment and subcarrier allocation $(\alpha_{i,j}^*, \beta_{j,k}^*)$ can be obtained in similar fashion. Finally, the dual problem is solved from the subgradient method. The detail steps of the solution are missing for simplicity and are similar to the solution proposed in Section 3.2.

5 | POWER OPTIMIZATION FOR GIVEN SUBCARRIER ALLOCATION AND RELAY ASSIGNMENT

The solution proposed in Sections 3 and 4 first find the power allocation for all the possible relay assignment and the subcarrier allocation and then, based on the obtained optimal power optimization, select the best relay selection and subcarrier assignment. This requires to solve NJK subproblems in each iteration of the subgradient update. In this section, we present a suboptimal scheme where the joint power allocation at the source and relay is obtained for the predefined $\alpha_{i,j}$ and $\beta_{j,k}$. The steps involved in the algorithm are listed as follows.

- Randomly allocate all the subcarriers such that the ith subcarrier is exclusively allocated to a unique user-relay pair (j, k). Thus, both $\alpha_{i,k}$ and $\beta_{j,k}$ are obtained.

- With obtained subcarrier and AR allocation, the optimization is similar to single-user single-AR power allocation problem, however with $J + 1$ independent power constraints instead of 2, ie,

$$\max_{p_i, u_{i,j}} \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} SR_{i,j,k}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} p_i \leq P_t, \tag{29}$$

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \alpha_{i,k} \beta_{j,k} u_{i,j} \leq Q_{t,j}. \quad \forall j$$

This problem can be solved using the similar dual technique in Section 3.2. However, note that now we need to find only N power variables instead of NKJ variables for power allocation at each step of the dual update.

6 | SIMULATION RESULTS

In this section, we present the simulation results to show the performance of the proposed schemes. We choose 6 tap channels taken from independent and identically distributed Gaussian random variables for all links and assume same noise variance at all nodes. For the analysis of the results, we compare the performance of following schemes:

OPT: this scheme includes the optimization of power loading over all subcarriers at both the BS and the AR node for the single-relay case, presented in Section 2.2.

Sub-OPT: in this algorithm, we consider the power optimization at the relay node only, whereas uniform power distribution is considered at the BS. Thus, it is the similar problem, as presented in Section 4, however for the single-user single-relay node case. The stepwise detail of the scheme is missing due to simplicity.

J-OPT: this refers to the joint optimization of relay selection, subcarrier allocation, and power allocation at all transmitting nodes, presented in Section 3.

Sub-OPT-I: it represents the solution presented in Section 4.

Sub-OPT-II: the solution with power optimization at the BS and all the relay nodes under fixed relay assignment and subcarrier allocation, as given in Section 5.

Non-OPT: this refers to the case with fixed subcarrier allocation, predefined relay selection, and equal power distribution among subcarriers at each transmitting nodes. Hence, for single-user single-relay case, this corresponds to uniform power allocation among all subcarriers at the 2 nodes.

Figure 3 presents the results for single-relay case where y-axis represents sum secrecy rate and x-axis represents total power budget. Same power budget is considered at the BS and AR, while we have set $N = 64$ and $N = 32$ for the upper and lower subplots, respectively. It can be clearly noted that the OPT scheme outperforms the remaining 2 schemes
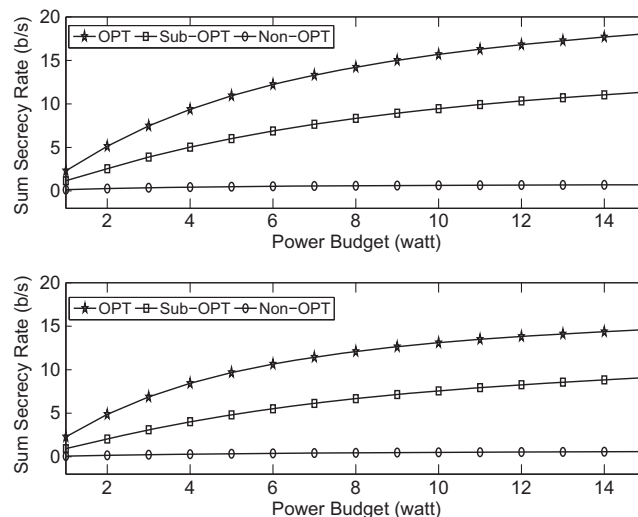

FIGURE 3 Sum secrecy rate vs power budget

and the Sub-OPT performs better than the Non-OPT, as presented in Figure 3. The performance gap between OPT and other candidates increases with the increase in the number of subcarriers and power budget. The better performance with increasing the number of subcarriers is due to the higher degree of freedom in power allocation. The increase in the power budget not only increase the sum secrecy rate for both OPT and Sub-OPT but also increases the gap. This is because of the fact that the OPT scheme efficiently distributes the available power budget among different subcarriers at the 2 transmitting nodes, whereas the Sub-OPT allocates power uniformly among subcarriers at the BS. Non-OPT does not provide secure communication as this scheme has zero sum secrecy rate, ie, the feasible solution does not exist with uniform power allocation. Hence, the resource optimization is mandatory for providing secure communication at the physical layer.

Next, in Figure 4, we show the convergence behavior of the dual variables for the 2 optimization schemes. Please note that the OPT involves 2 dual variables, whereas the Sub-OPT has a single dual variable. It can be observed that both the schemes converge within acceptable number of iterations. Furthermore, it is noted that the OPT provides higher performance at the cost of few more number of iterations for convergence. On the other hand, the Sub-OPT provides much better performance over the Non-OPT without requiring high burden of time consumption in terms of number of iterations.

In Figure 5, we consider multiuser multirelay scenario with $J = 4$ and $K = 12$. Similar to Figure 3, the same available power budget is assumed at all nodes and results are obtained with $N = 32$ as well as for $N = 64$. It can be seen from Figure 5 that there is a clear gap between the J-OPT and the other schemes while Sub-OPT-I outperforms Sub-OPT-II. Increasing
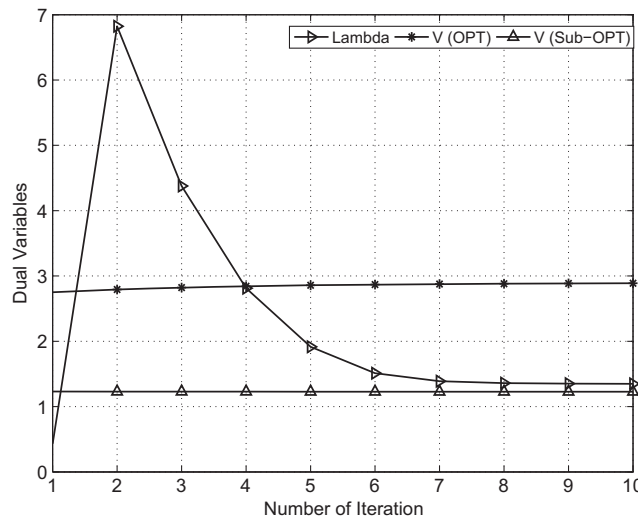

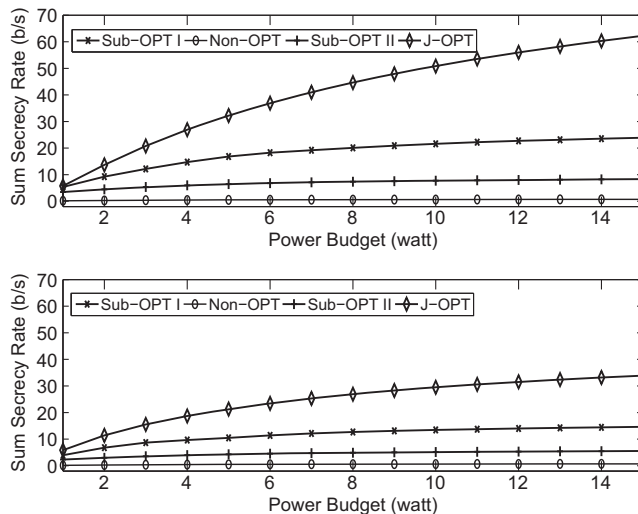
FIGURE 4 Convergence rate

FIGURE 5 Sum secrecy rate vs power budget

the number of subcarriers increases the degree of flexibility of power allocation, which results in increasing secrecy rate of all schemes. It is also interesting to note that the enhancement in performance of J-OPT scheme with increasing N is higher than the other schemes.

For Figures 6 and 7, we have taken a single realization of Gaussian random channels to show the possible effects of adding a new relay or user in the system. Figure 6 shows the impact of varying number of relays from 1 to 4, with $P_t = Q_t = 7$, $K = 12$ and $N = 64$. We can see similar trends as in Figure 5. Increasing the number of relays provides enhanced performance for all schemes. However, the percentage increase of the sum secrecy rate of J-OPT and Sub-OPT-I is much more than that of the other 2 players. This is because both schemes involve optimizing relay selection and subcarrier assignment, whereas the other 2 schemes use fixed relay selection and subcarrier assignment. Last but not least, the rate of increase in secrecy rate is more from $J = 1$ to $J = 3$ and becomes a bit low from $J = 3$ to $J = 4$. This is because initial addition of relays provides a higher freedom in resource allocation. For a more closer look into exact values, the results are also depicted in Table 3.

Finally, to complete the analysis, we check the sum secrecy rate for different number of users with fixed number of relays. The results are plotted in Figure 7 with $N = 64$, $PT = QT = 7$, and $J = 4$. Again, the superiority of J-OPT over all competing candidates is clear. In J-OPT, increasing the number of users enhances the performance if the channel gains of new user are better compared to old users. This is due to the fact that higher number of users provides better channel
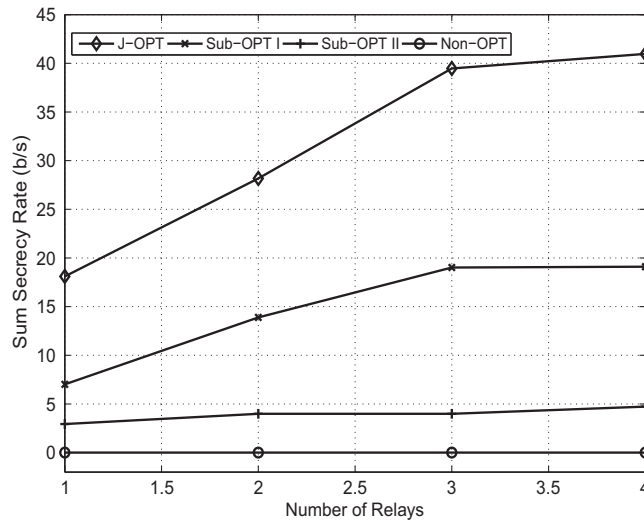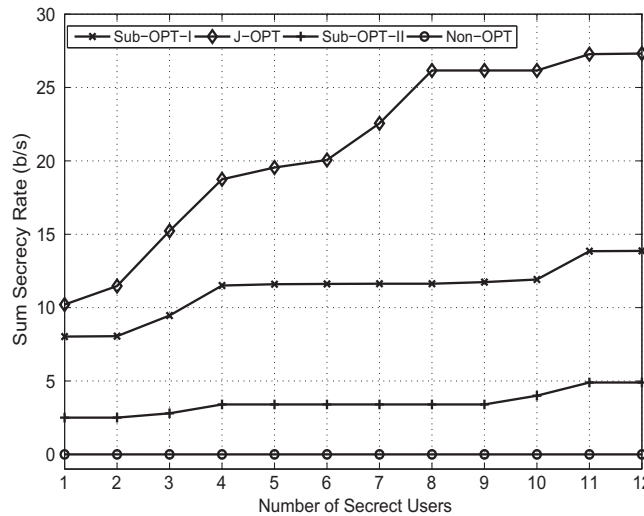

FIGURE 6 Sum secrecy rate vs number of relays

FIGURE 7 Sum secrecy rate vs number of users

TABLE 3 Sum secrecy rate for different number of relays

Number of Relays	J-OPT	Sub-OPT I	Sub-OPT II	Non-OPT
1	18.10	7.01	2.94	0
2	28.17	13.88	3.97	0
3	39.47	19.01	3.99	0
4	40.97	19.10	4.73	0

conditions and higher flexibility in power allocation. The J-OPT considers all parameters jointly; hence, we observe a significant performance gap increase with increasing the number of users.

7 | CONCLUSION

This paper considered resource allocation problem to enhance PLS in AR-assisted wireless networks. Joint optimization problem of power allocation at different transmitting nodes, relay assignment, and the subcarrier allocation has been studied. For practical reasons, separate power constraint was considered at the BS and each relaying node. A dual decomposition framework was adopted to find an efficient solution for subcarriers allocation, relays assignment, and power loading over all subcarriers. The target was to maximize the sum secrecy rate of the system. Furthermore, suboptimal schemes were also presented. Simulation results validated the performance of all proposed schemes. Joint optimization and suboptimal schemes outperformed the trivial solutions. It was observed that the gain of the proposed joint optimization solution increases with the increases in the number of subcarriers, number of relays, and total power budget.

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