Thresholds Optimization for One-Bit Feedback Multi-User Scheduling

Mohammed Hafez, Ahmed El Shafie, Mohammed Shaqfeh, Tamer Khattab, Hussein Alnuweiri, and Hüseyin Arslan

***Abstract*—We propose a new one-bit feedback scheme with scheduling decision based on the maximum expected weighted rate. We show the concavity of the 2-user case and provide the optimal solution which achieves the maximum weighted rate of the users. For the general asymmetric *M*-user case, we provide a heuristic method to achieve the maximum expected weighted rate. We show that the sum rate of our proposed scheme is very close to the sum rate of the full channel state information case, which is the upper bound performance.**

***Index Terms*—Diversity, feedback, multiuser, massive MIMO, scheduling.**

I. Introduction

CHANNEL-AWARE adaptive transmission techniques [1] and dynamic resource allocation, using opportunistic scheduling, are applied practically in modern communication systems to maintain good performance under dynamic environment. The underlying objective of these schemes is exploiting the fading wireless channels when they are at their peak conditions to achieve significant capacity gains [2]. Explicit training sequences (i.e., pilot signals) are used in current wireless communication systems to enable the receivers to measure the instantaneous channel conditions so that coherent detection of the transmitted signals can be applied [3]. In opportunistic scheduling schemes, channel state information (CSI) of all back-logged mobile users in the network should be known at the central scheduler, i.e., base station (BST). The mobile terminals inform the central scheduler about their CSI using explicit feedback messages. The main drawback is that the CSI feedback consumes a considerable portion of the total air-link resources. Moreover, with the consideration of massive multiple-input multiple-output (MIMO) as an enabling technology for 5G networks, the associated pilot contamination problem rises as a performance limiter [4]. Massive MIMO requires perfect CSI knowledge to achieve optimal performance. This would require both high amount of training overhead and feedback which usually increases with the increase of the number of antennas. Based on some experimental studies, it is found that the channel experiences a sparse behavior. This sparse structure of the channel allows the application of some compressed sensing techniques to reduce training overhead. On the other hand, the feedback load can be onerous which makes the adoption of a reduced feedback structure desirable. Consequently, there is a need to develop new transmission technologies with reduced CSI feedback from the users to the BST.

The main technical challenge in the design of a reduced-feedback multi-user scheduling scheme is maintaining the expected capacity gain due to multi-user diversity. This is an important research topic that is seldom investigated thoroughly in [5]. Extensive surveys on feedback reduction methods are provided in [6]. The switched-diversity multi-user scheduling schemes are considered as reduced-feedback schemes [7], [8]. While [7] considers the long-term channel statistical information in their optimization approach, [8] is based on one-bit feedback per user per CSI with pre-determined priority order of the users. The CSI feedback is controlled by a threshold on the channel condition to decide the one-bit feedback to be sent to the BST.

In this letter, we propose a new approach to the one-bit feedback problem in which the user scheduling is based on the maximum expected reward, i.e., the weighted rate, conditional on the one-bit feedback sent from all users. We optimize the feedback thresholds of each user for this scheme and show that the performance of this limited-feedback scenario is not far from the full-CSI feedback case. We provide analytic solutions for the 2-user case. For the general case of M asymmetric users, we provide a heuristic method to achieve the maximum expected weighted rate.

II. System Model

We consider a downlink system with a single BST and M users. All nodes are equipped with single antennas. We assume a block-fading channel model where the channels remain quasi-static during the coherence time and changes from one coherence time duration to another. The channels are assumed independent from one link to another, as suggested in [4] and [9], and are also assumed to be **non-reciprocal** which implies that the channel from node A to node B is not equal to the channel from node B to node A. The BST is not aware of the CSI over each individual channel block. However, it knows the statistical information of the channels of all users, which is presented by the probability density function (PDF) of the achievable rate r_i, denoted by $f_i(r)$ for the channel of user i. The BST calculates the optimized threshold values and forwards them to the users. At each transmission time interval (TTI), each user compares its channel condition with the provided threshold. A one-bit feedback signal from each user per channel block is used to aid the BST in selecting which user

Manuscript received December 6, 2017; revised January 13, 2018; accepted February 8, 2018. Date of publication February 15, 2018; date of current version August 21, 2018. This work was supported by the Qatar National Research Fund (a member of the Qatar Foundation, QF) under Grant NPRP 7-923-2-344. The statements made herein are the sole responsibility of the authors. The associate editor coordinating the review of this paper and approving it for publication was S. Andreev. *(Corresponding author: Ahmed El Shafie.)*

M. Hafez is with the Electrical Engineering Department, University of South Florida, Tampa, FL 33620-9951 USA (e-mail: mhafez@mail.usf.edu).

A. El Shafie is with the Electrical Engineering Department, University of Texas at Dallas, Richardson, TX 75080 USA (e-mail: ahmed.elshafie@utdallas.edu).

M. Shaqfeh and H. Alnuweiri are with the Electrical and Computer Engineering Department, Texas A&M University at Qatar, Doha, Qatar (e-mail: mohammad.shaqfeh@qatar.tamu.edu; hussein.alnuweiri@qatar.tamu.edu).

T. Khattab is with the Electrical Engineering Department, Qatar University, Doha, Qatar (e-mail: tkhattab@ieee.org).

H. Arslan is with the Electrical Engineering Department, University of South Florida, Tampa, FL 33620-9951 USA, and also with the Department of Electrical and Electronics Engineering, Istanbul Medipol University, 34810 Istanbul, Turkey (e-mail: arslan@usf.edu).

Digital Object Identifier 10.1109/LWC.2018.2806453

will be scheduled in a given channel block to receive data. The one-bit feedback of a user indicates whether its channel, in a given channel block, is above or below a predetermined threshold of the achievable rate, denoted by r_i. Here, the single bit feedback to the BST represents the condition of one of the available resources (i.e., the user needs to feedback a single bit for each available resource).

The objective of the scheduling task is to select the user with the highest weighted rate to receive data, where the weight of a user, denoted by μ_i for user i, is predetermined before the actual operation (based on quality-of-service (QoS) requirements for each user) and hence is not subject to optimization. Since the BST does not have exact knowledge of the instantaneous CSI of all users, it incorporates its knowledge of the PDFs of the users' channels and the received one-bit feedback from each user in the scheduling decision. Therefore, in every channel block, the BST schedules the user with the highest weighted *expected achievable rate* conditioned on the received feedback information from all users, $\mu_i \mathbb{E}[R_i(k)|b_1, b_2, \ldots, b_M]$, where $\mathbb{E}[\cdot]$ denotes the expected value of the argument, $R_i(k)$ denotes the achievable rate of user i during the channel block k, and b_i is the one-bit feedback from user i.

We optimize the feedback thresholds of the users to maximize the weighted-sum rates of all users, denoted Φ

$$\Phi = \sum_{i=1}^{M} \mu_i \tilde{R}_i, \tag{1}$$

where $\tilde{R}_i$ is the expected rate of each user (i.e., averaged across channel blocks). Therefore, we can write the main optimization problem as

$$\{r_1^*, \ldots, r_M^*\} = \underset{\{r_1,\ldots,r_M\}}{\arg\max}\, \Phi. \tag{2}$$

Let us define two quantities for each user, $R_i^+ = \mathbb{E}[R_i|R_i > r_i]$ and $R_i^- = \mathbb{E}[R_i|R_i < r_i]$. These can be obtained using:

$$R_i^+ = \frac{\int_{r_i}^{\infty} r f_i(r) dr}{1 - F_i(r_i)}, \;\; R_i^- = \frac{\int_0^{r_i} r f_i(r) dr}{F_i(r_i)} \tag{3}$$

where $F_i(r)$ is the cumulative distribution function (CDF) of the achievable rate of the channel of user i. Moreover, we need to define two quantities to study the effect of the feedback of one user on the scheduling probability of another user, $\Omega_{ij}^+ = \Pr\{\mu_i R_i > \mu_j R_j | R_i > r_i\}$ and $\Omega_{ij}^- = \Pr\{\mu_i R_i > \mu_j R_j | R_i < r_i\}$, where $i \neq j$. These can be obtained using

$$\Omega_{ij}^x = \begin{cases} 1 & \text{if} \quad \mu_j R_j^+ < \mu_i R_i^x \\ F_j(r_j) & \text{if} \quad \mu_j R_j^- < \mu_i R_i^x < \mu_j R_j^+ \\ 0 & \text{if} \quad \mu_i R_i^x < \mu_j R_j^- \end{cases}, \tag{4}$$

where $x \in \{-, +\}$. Hence, $\tilde{R}_i$ is given by

$$\tilde{R}_i = R_i^+[1 - F(r_i)] \prod_{j \neq i} \Omega_{ij}^+ + R_i^- F(r_i) \prod_{j \neq i} \Omega_{ij}^-. \tag{5}$$

In our proposed scheme, the BST sends a single pilot signal to all users. Then, each user checks if its channel is higher or lower than the threshold. Afterwards, each user sends one-pilot binary signal to inform the BST if its channel is above or below the threshold. After the BST collects the statuses of all users, it sends the index of the user selected for data reception. Since we have M users, the BST requires $\lceil \log_2(M) \rceil$ bits to inform the users about the user selected for data reception. We emphasize here that we **do not** assume channel reciprocity. If the channel is reciprocal, then the users can send a known pilot signal over one symbol duration to the BST to estimate each channel link. Then, the BST selects the best link and there is **no need** to feed back the channels from the users to BST, which requires a significant overhead. Assuming a bandwidth of W Hz, the symbol duration is $1/W$ seconds. Hence, our proposed scheme consumes in total $\frac{M+1+\lceil \log_2(M) \rceil}{W}$ seconds where $1/W$ is the duration of the pilot signal from the BST to all users, M/W is the duration for all users to send their channel status to the BST (where each user's pilot signal consumes $1/W$ seconds), and $\lceil \log_2(M) \rceil$ is the number of bits required to announce which user has been selected for data reception. On the other hand, since the channel is not reciprocal, in the conventional schemes, after the BST sends a pilot signal, each user estimates its channel. Then, each user feeds back its CSI to the BST, which consumes Mf/W with f denoting the number of quantization bits (i.e., number of bits used to quantize each user/link CSI) and f/W is the time spent to send the CSI of **one** user. This feedback duration, Mf/W, is significantly increasing with the number of feedback bits (which increases the accuracy of feeding back the channel) and the number of user.

III. Optimized Thresholds

In this section, we will investigate the problem of finding the optimum thresholds of each user.[1] We start with the 2-user case. Then, we extend the analysis to the general M-user case.

A. 2-User System

Based on the construction of the system, and the structure of (4), we have six different formats of Φ, each of them represents a different combination of the values of (5).

Consider the case $\mu_1 R_1^+ > \mu_2 R_2^+$. This will limit the number of available formats of Φ into three, which are shown in (7) at the top of the next page. Also, we can rewrite (7)c) as in (8) and (9), as shown at the top of the next page.

Proposition 1: A local peak for the value of Φ can be found in the region where the selected thresholds meet one of the following conditions

$$\mu_1 R_1^+ > \mu_2 R_2^+ > \mu_1 R_1^- > \mu_2 R_2^-, \tag{6a}$$

$$\mu_2 R_2^+ > \mu_1 R_1^+ > \mu_2 R_2^- > \mu_1 R_1^-. \tag{6b}$$

Proof: From (8), it is shown that (7c) is always larger than (7a), as the value of $(\mu_2 R_2^+ - \mu_1 R_1^-)$ is always positive. This is also true for (9) and (7b), where $(\mu_1 R_1^- - \mu_2 R_2^-)$ is always positive. Similarly, we can show that the case of $(\mu_1 R_1^+ < \mu_2 R_2^+)$ provides the same results. ■

Based on Proposition 1, the search for the optimum thresholds has been limited into two regions instead of six.

Proposition 2: The optimum thresholds for condition (6a) are given by

$$r_1^* = \frac{\mu_2}{\mu_1} R_2^+, \;\; r_2^* = \frac{\mu_1}{\mu_2} R_1^- \tag{10}$$

and similarly for the case of (6b),

$$r_1^* = \frac{\mu_2}{\mu_1} R_2^-, \;\; r_2^* = \frac{\mu_1}{\mu_2} R_1^+. \tag{11}$$

Proof: See Appendix A. ■

B. M-User System

In the M-user case, the optimization problem solution for finding the global peak is not feasible.[2] Hence, we provide a heuristic approach to find the solution. Following

[1] The optimization process solely depends on the statistical information of the channel, which does not change as frequently as the instantaneous CSI.

[2] With the increase in the number of users, the number of peaks gets overwhelmingly large. The number of possible formats of Φ as a function of the number of users is $3M!$.

$$\Phi = \begin{cases} \mu_1 \int_0^\infty r f_1(r) dr & (\mu_2 R_2^+ < \mu_1 R_1^-) \quad (a) \\ \mu_1 R_1^+[1 - F_1(r_1)] + \mu_2 F_1(r_1) \int_0^\infty r f_2(r) dr & (\mu_1 R_1^- < \mu_2 R_2^-) \quad (b) \\ \mu_1 R_1^+[1 - F_1(r_1)] + \mu_1 R_1^- F_1(r_1) F_2(r_2) + \mu_2 R_2^+ F_1(r_1)[1 - F_2(r_2)] & (\mu_2 R_2^- < \mu_1 R_1^- < \mu_2 R_2^+) \quad (c) \end{cases} \quad (7)$$

$$\Phi = \mu_1 \int_0^\infty r f_1(r) dr + \left[\mu_2 R_2^+ - \mu_1 R_1^-\right] F_1(r_1)[1 - F_2(r_2)] \quad (8)$$

$$\Phi = \mu_1 R_1^+[1 - F_1(r_1)] + \mu_2 F_1(r_1) \int_0^\infty r f_2(r) dr + \left[\mu_1 R_1^- - \mu_2 R_2^-\right] F_1(r_1) F_2(r_2) \quad (9)$$

the same argument of the 2-user case, the condition in (6) can be extended to

$$\mu_1 R_1^+ > \mu_2 R_2^+ > \cdots > \mu_M R_M^+ > \mu_1 R_1^- > \mu_2 R_2^- > \cdots > \mu_M R_M^-. \quad (12)$$

Hence, the number of the local peaks would be $M!$. In general, for any number of users, M, the formula of the rate based on the condition in (12) is given by

$$\tilde{R}_1 = R_1^+[1 - F_1(r_1)] + R_1^- \prod_{k=1}^{M} F_k(r_k),$$

$$\tilde{R}_i = R_i^+[1 - F_i(r_i)] \prod_{k=1}^{i-1} F_k(r_k) \qquad \forall i > 1. \quad (13)$$

Hence,

$$\Phi = \sum_{j=1}^{M} \left[\mu_j R_j^+[1 - F_j(r_j)] \prod_{n=0}^{j-1} F_n(r_n) \right] + \mu_1 R_1^- \prod_{n=1}^{M} F_n(r_n). \quad (14)$$

This can be reformulated into $M!$ different formulas as

$$\Phi_m = \sum_{a=0}^{m-1} \left[\mu_a R_a^+[1 - F_a(r_a)] \prod_{b=0}^{a-1} F_b(r_b) \right] + \mu_m g_m \prod_{b=0}^{m-1} F_b(r_b) + \sum_{a=m+1}^{M} \left[\left(\mu_a R_a^+ - \mu_m R_m^-\right)[1 - F_a(r_a)] \prod_{b=0}^{a-1} F_b(r_b) \right] + \left(\mu_1 R_1^- - \mu_m R_m^-\right) \prod_{b=0}^{M} F_b(r_b), \quad (15)$$

where $F_0(r_0) = 1$, $\mu_0 = 0$, and $m = 1, 2, \ldots, M!$. Each of these formulas represents a region of thresholds where a local peak exists. To find the optimum thresholds, we equate the first partial derivatives of Φ with respect to r to zero. That is,

$$\frac{\partial \Phi_m}{\partial r_i^*} = 0, \quad (16)$$

and based on (13), we find that the optimum thresholds in a particular region take the following values:

$$r_M^* = \frac{\mu_1}{\mu_M} R_1^-,$$

$$r_{i \neq \{1,M\}}^* = \frac{\mu_{i+1}}{\mu_i} \left[\hat{r}_{i+1}^* F_{i+1}\left(\hat{r}_{i+1}^*\right) + R_{i+1}^+\left(1 + F_{i+1}\left(\hat{r}_{i+1}^*\right)\right) \right],$$

$$r_1^* = \frac{\mu_2\left[\hat{r}_2^* F_2\left(\hat{r}_2^*\right) + R_2^+\left(1 + F_2\left(\hat{r}_2^*\right)\right)\right] - \mu_M \hat{r}_M^* \prod_{k=2}^{M} F_k\left(\hat{r}_k^*\right)}{\mu_1\left[1 + \prod_{k=2}^{M} F_k\left(\hat{r}_k^*\right)\right]}. \quad (17)$$

It is noticeable that the corresponding values of the optimum thresholds recursively depend on each other. Hence, we find their values using numerical methods. In our case, the bisection algorithm is used to calculate the optimum thresholds for one of the multiple local-optimas. The selection of that local-optima can be realized using one of the following approaches.

1) Brute-Force Search: For this case, for each of the $M!$ regions, the optimum thresholds should be calculated, and compare the values of Φ_m to get the maximum value.

2) Threshold-Independent Function: This approach is based on defining a certain indicator utility function $g_i(\mu_i, f_i(r))$, which is independent of the thresholds. Using this method, we have only M calculations for the values of the functions g_i, then we order the users based on their corresponding values, and finally calculate the optimum thresholds only once.

3) Random Selection: An alternative approach is to randomly select one of the $M!$ regions, calculate the optimum thresholds, and decide based on these thresholds. This approach will have minimal calculations requirements, and we will show that it has a negligible performance loss.

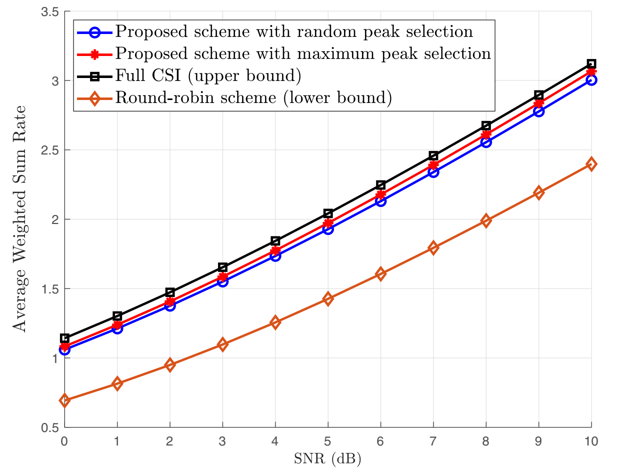

Fig. 1. Comparing the average weighted sum rate of our proposed one-bit feedback scheme with the full-CSI case (upper bound performance).

IV. Numerical Results and Conclusions

Each channel coefficient is modeled as a complex Gaussian circularly symmetric random variable with zero mean and unit variance [4], [9]. The channel coefficients are compared to the pre-calculated thresholds and either positive (i.e., '1') or negative (i.e., '0') feedback is sent to the BST. The BST selects the user with the highest achievable rate among all users with positive feedback, and assigns it the respective resource. The system is simulated for five different users. Even though there is no limitation on the number of users that can be served under such system, in order to limit the complexity of the optimization problem, the BST can set a limit on the number of users for a certain group of resources. We assume that $\mu_i = [1.1, 1.05, 1, 0.95, 0.9]$. The analysis here considers a single resource to be assigned to one of the users. In the

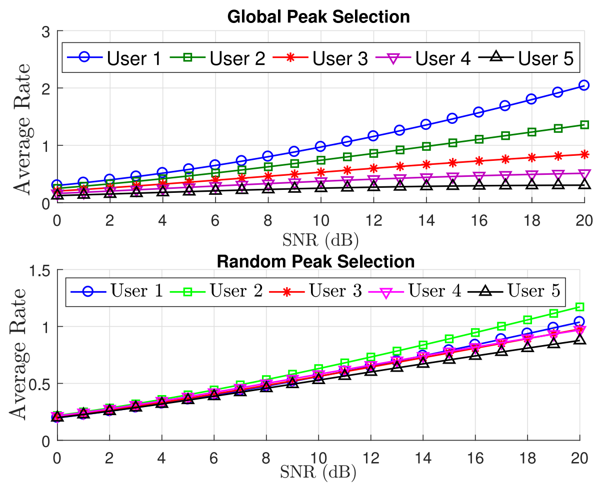

Fig. 2. Change of average rates versus average SNR under two different optimization approaches.

numerical simulations, we wanted to show that even without considering the feedback cost of any of the schemes, which is much higher in the global CSI compared to our proposed scheme, our scheme performs closely to the global CSI scheme.

Fig. 1 shows that the average weighted rate of our proposed one-bit feedback system is comparable to the full-CSI case (upper bound performance), and has a good gap over the round-robin scheduling (lower bound performance). Moreover, we show the performance of randomly selecting any peak. We can find that randomly selecting any local peak has a negligible effect on the system performance, which can be a good choice to avoid the complexity of finding the global peak.

Fig. 2 shows average rate for each user. The figure includes two different cases where the top and bottom subfigures show the global peak selection and random peak selection cases, respectively. We can notice that the rates are close to each other in the low signal-to-noise ratio (SNR) levels, and the gaps between them become wider as the SNR increases. It is noticeable that the QoS requirements affect the way the sum rate is maximized. Users with higher priorities are allocated more frequently which gives them higher rates at better channel conditions. On the other hand, users with lower priorities maintain the same rates regardless of the channel conditions. For the random peak selection case, the change of user ordering for each peak results in a change of their respective thresholds. The change of the threshold values provides a higher probability of scheduling for low rate users. For this reason, the rates are more close to each other which provides a better fairness performance compared to the global peak selection.

Fairness can also be achieved through an adaptive priority assignment. The priority of a user would increase if it is not assigned a resource for a given time interval; or if a certain fairness objective is desired. For the latter case, the priority weights can be optimized based on such fairness objective. The study of optimizing the priority weights is out of the scope of this letter.

APPENDIX A
OPTIMAL THRESHOLD FOR 2-USER CASE

To find the optimum thresholds for the 2-user case, we solve the following constrained optimization problem:

$$\max_{r_i}.\ \Phi,\ \text{s.t. } 0 \le r_i,\ \ \forall i \in \{1,2\}. \tag{18}$$

The maximum of Φ is obtained using (6) under the constraint that $\mu_2 R_2^- < \mu_1 R_1^- < \mu_2 R_2^+ < \mu_1 R_1^+$. Therefore, the optimization problem can be stated as follows:

$$\max_{r_i}.\ \left[\mu_2 F_1(r_1)\int_{r_2}^{\infty} r f_2(r)dr - \mu_1[1-F_2(r_2)]\int_0^{r_1} r f_1(r)dr\right],$$
$$\text{s.t. } 0 \le r_i \forall i \in \{1,2\},\ \mu_2 R_2^- < \mu_1 R_1^- < \mu_2 R_2^+ \le \mu_1 R_1^+. \tag{19}$$

For a fixed (given) r_2, the optimization problem is given by

$$\max_{r_i}.\ \left[K_2 F_1(r_1) - K_1\int_0^{r_1} r f_1(r)dr\right],$$
$$\text{s.t. } 0 \le r_i \le 1 \forall i \in \{1,2\},\ \mu_2 R_2^- < \mu_1 R_1^- < \mu_2 R_2^+ < \mu_1 R_1^+. \tag{20}$$

The first derivative is given by

$$\frac{\delta \Phi}{\delta r_1} = (K_2 - K_1 r_1) f_1(r_1), \tag{21}$$

where $K_2 = \mu_2 \int_{r_2}^{\infty} r f_2(r)dr$, $K_1 = \mu_1 \overline{F_2(r_2)}$, and $\overline{\mathcal{X}} = 1 - \mathcal{X}$. If $r_1 > K_2/K_1$, the derivative is negative; hence, the objective function is monotonically decreasing with r_1. This means that the optimal solution for a fixed r_2 is attained when we set r_1 to its lowest feasible value. This value is obtained from the constraint $\mu_2 R_2^- < \mu_1 R_1^- < \mu_2 R_2^+$. Similarly, If $r_1 < K_2/K_1$, the derivative is positive; hence, the optimal solution for a fixed r_2 is attained when we set r_1 to its highest feasible value.

The second derivative is given by

$$\frac{\delta^2 \Phi}{\delta r_1^2} = -K_1 f_1(r_1) + (K_2 - K_1 r_1)\frac{\delta f_1(r_1)}{\delta r_1}. \tag{22}$$

If $(K_2 - K_1 r_1)\frac{\delta f_1(r_1)}{\delta r_1} \le 0$, then the objective function is concave. Setting the first derivative to zero, we get $r_1^* = K_2/K_1$. This condition maintains the concavity of the problem and it is the optimal solution if and only if it satisfies the constraints. We note that $r_1^* > (\mu_2 r_2)/\mu_1$.

In this case, we can convert the constraint from (6) into three constraints. That is,

$$\mu_2 R_2^+ - \mu_1 R_1^- > 0 \leftrightarrow (\mu_2 R_2^+ - \mu_1 R_1^-) F_1(r_1)[1 - F_2(r_2)] > 0,$$
$$\mu_1 R_1^- > \mu_2 R_2^-,\ \mu_1 R_1^+ > \mu_2 R_2^+. \tag{23}$$

Finally, it can be easily verified that the optimal solution satisfies the three constraints.

REFERENCES

[1] A. Svensson, G. E. Ien, M. S. Alouini, and S. Sampei, "Special issue on adaptive modulation and transmission in wireless systems," *Proc. IEEE*, vol. 95, no. 12, pp. 2269–2273, Dec. 2007.
[2] X. Wang, G. B. Giannakis, and A. G. Marques, "A unified approach to QoS-guaranteed scheduling for channel-adaptive wireless networks," *Proc. IEEE*, vol. 95, no. 12, pp. 2410–2431, Dec. 2007.
[3] A. Dual-Hallen, "Fading channel prediction for mobile radio adaptive transmission systems," *Proc. IEEE*, vol. 95, no. 12, pp. 2299–2313, Dec. 2007.
[4] L. Lu, G. Y. Li, L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, Oct. 2014.
[5] V. Raghavan, J. J. Choi, and D. J. Love, "Design guidelines for limited feedback in the spatially correlated broadcast channel," *IEEE Trans. Commun.*, vol. 63, no. 7, pp. 2524–2540, Jul. 2015.
[6] D. J. Love *et al.*, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.
[7] V. Raghavan, S. V. Hanly, and V. V. Veeravalli, "Statistical beamforming on the Grassmann manifold for the two-user broadcast channel," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6464–6489, Oct. 2013.
[8] M. Shaqfeh, H. Alnuweiri, and M.-S. Alouini, "Multiuser switched diversity scheduling schemes," *IEEE Trans. Commun.*, vol. 60, no. 9, pp. 2499–2510, Sep. 2012.
[9] K. Zheng, S. Ou, and X. Yin, "Massive MIMO channel models: A survey," *Int. J. Antennas Propag.*, vol. 2014, p. 10, Jan. 2014.