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Technical assessment of solar energy storage investments with recommender system-enhanced quantum picture fuzzy rough sets

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ABSTRACT

The performance of solar energy storage projects should be improved by taking appropriate actions. However, there are very different criteria that affect the performance of these investments. Therefore, businesses need to focus on more important criteria to use the budget effectively and efficiently. This situation increases the need for a priority analysis for performance indicators of solar energy storage investments. Accordingly, the purpose of this study is to make evaluation for the technical assessment of solar energy storage investments. In this scope, a new four-stage model is introduced by considering different decision-making techniques and fuzzy sets. The first stage is related to the prioritizing the experts with artificial intelligence (AI)-based decision-making method. Secondly, the missing evaluations of solar energy storage investments are estimated with expert recommender system. In the following part, the criteria for the technical assessment of solar energy storage investments are weighted by quantum picture fuzzy rough sets (QPFRS) adopted M−SWARA. The final stage consists of ranking the solar energy storage alternatives with QPFR-VIKOR. The main contribution of this study is the generation of the decision matrix by the help of AI. This situation gives an opportunity to calculate the significance weights of the experts. Therefore, the analysis results can be more reliable and coherent. It is concluded that battery capacity is the most critical factor for the technical assessment of solar energy storage investments. On the other hand, pumped hydro for mechanic energy is found as the most significant solar energy storage alternative. Governments should provide the necessary incentives for the development of high-capacity battery technologies. In this context, tax reductions can be provided to companies that invest in production technologies. This contributes to the cost efficiency of businesses.

1. Introduction

Solar energy projects provide many benefits in terms of environmental, economic, and social aspects. However, there are some issues that hinder the development of these projects. One of the biggest drawbacks in solar energy projects is the incompatibility between energy production and consumption. The main reason for this is that solar energy can only be obtained at certain times of the day [1]. Therefore, energy storage processes are seriously needed in solar energy projects. Energy storage systems store excess produced energy and enable it to be used later when needed. On the other hand, fluctuations in energy production can be prevented thanks to effective energy storage processes. This enables a more reliable energy source to be obtained. One of the benefits of solar energy storage investments is cost savings [2]. This contributes significantly to the financial success of businesses. Similarly, thanks to effective storage systems, dependence on fossil fuels can be reduced. This contributes to reducing many environmental problems such as carbon emissions.

Since solar energy storage projects are of critical importance, necessary measures must be taken to increase the effectiveness of these investments. Energy efficiency is necessary to increase the economic performance of these projects. This significantly helps reduce energy loss [3]. Energy losses can be minimized by developing an efficient system. This situation allows more energy to be used. Increasing the

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battery capacity is also critical in improving the performance of these systems. This allows solar energy storage systems to store more energy. Thus, it is possible to ensure the continuity of energy supply. Similarly, a larger portion of the energy produced from solar energy can be stored thanks to batteries with larger capacity [4]. Thus, it is possible to increase the efficiency in energy production processes. Establishing energy storage facilities in a limited area is necessary to increase the performance of this storage system. The biggest advantage of this situation is that it ensures the most efficient use of the available space.

Necessary actions need to be taken to increase the performance of solar energy storage projects. However, there are very different criteria that affect the performance of these investments. On the other hand, each action taken leads to an increase in the operational costs of businesses. Therefore, businesses making too many improvements causes costs to increase excessively. Therefore, businesses need to focus on more important criteria rather than too many improvements to avoid financial problems. This helps avoid incurring very high costs while taking precautions. Therefore, it is necessary to determine the most important factors affecting the performance of solar energy storage projects. In addition to this issue, the companies have limited financial and human sources. These limited sources should be used effectively. To achieve this objective, the most critical determinants should be identified. With the help of this issue, the most effective strategies can be determined and implemented. In this case, analyzes are needed to determine the importance weight of the criteria. Most of the studies in the literature highlight the significance of the indicators. Nevertheless, studies focusing on this subject in the literature are quite limited. This situation is determined as an important gap in the literature. On the other side, decision-making models can also be considered in this context. Nonetheless, energy storage effectiveness is a very complex issue, and currently generated models may not be successful to analyze this complex problem. Hence, to make a sensitive evaluation, a novel decision-making model should be proposed.

Accordingly, in this study, it is aimed to make evaluation for the technical assessment of solar energy storage investments. In this context, the main research question is to identify which solar energy storage alternatives are more effective and efficient. For this purpose, a new four-stage model is generated by considering different decision-making techniques and fuzzy sets. The first stage is related to the prioritizing the experts with AI-based decision-making method. Secondly, the missing evaluations of solar energy storage investments are estimated with expert recommender system. In the following part, the criteria for the technical assessment of solar energy storage investments are weighted by QPFRS M−SWARA. The final stage consists of ranking the solar energy storage alternatives with QPFR-VIKOR. The main motivation of this study is the necessity for a comprehensive evaluation to define the most important factors affecting the performance of solar energy storage projects. The main reason is that there is no consensus in the literature about which determinants are the most crucial for the performance improvements of solar energy projects. Owing to this study, appropriate investment strategies can be implemented for solar energy investors. In this process, the companies can implement effective investment strategies without having extra costs. Hence, the performance of these projects can be improved. Due to this situation, a priority analysis should be conducted to define the most effective investment strategies. To find the most critical determinants, decision-making models can be taken into consideration. However, existing models can be criticized due to many different reasons. For instance, it is claimed that taking the direct average of experts' opinions is not realistic. The main reason for this is that experts have different work experiences and education levels. Therefore, it is thought that existing models may not be sufficient to determine the most effective investment strategies for solar energy storage projects. This situation explains that there is a strong need for a novel decision-making model for the subject of energy storage investment performance. In this context, in this study, a new model is developed in which the importance weights of experts are calculated.

The main contribution of this study is that effective investment strategies can be identified for the technical assessment of solar energy storage projects by establishing a new model. The main theoretical contribution of this study is that appropriate actions can be taken in an efficient way to improve energy storage performance. Thus, more environment-friendly energy production can be increased. Similarly, countries can have energy independence with the help of solar energy project improvements. On the other side, the superiorities of the proposed model are underlined below. (i) M−SWARA methodology is considered to compute the weights of the criteria. This new methodology is created by implementing some improvements to SWARA. Owing to this action, causality relationship between the determinants can be defined. The main indicators of the technical assessment of solar energy storage investments can have a causal impact on each other. Hence, M−SWARA methodology helps to reach more appropriate findings. (ii) The generation of the decision matrix by the help of AI provides some superiorities to the proposed model. This situation gives an opportunity to calculate the significance weights of the experts. In other words, the evaluations of these people can be taken into consideration with the coefficients based on their demographic issues. Therefore, the analysis results can be more reliable.

Literature evaluation is conducted in the second section. The proposed model and main findings are highlighted in the next parts. The discussions and conclusions are denoted in the final sections.

2. Literature review

The biggest disadvantage of solar energy is its intermittency. Solar energy storage systems are needed to reduce this problem [5]. Energy efficiency in the storage process is one of the important criteria in increasing the efficiency of solar energy storage investments. In case of high energy efficiency in the storage process, it is possible to utilize solar energy more. Liu et al. [6] stated that calcium-based material should be designed to store solar energy more efficiently. Chen and Fang [7] argued that lithium-ion batteries used in the storage of solar energy are a more efficient and safe method for storing energy. Cao et al. [8] concluded that the storage of solar energy is hampered by geographical and weather conditions. For this reason, thermal energy storage technology will be a more efficient way to store solar energy. Xiao et al. [9] indicated that hybrid energy systems consisting of wind and solar energy are more cost-effective energy storage methods.

Battery capacity is among the important criteria for increasing the effectiveness of solar energy storage investments. Increasing the battery capacity means increasing the energy to be stored. This situation allows the non-consumed part of the energy to be used later. By increasing the battery capacity, energy efficiency is ensured. Yoo and Ha [10] showed that the BESS system, referred to as battery energy storage systems, will be a solution for storing intermittent renewable energy sources such as wind and solar. However, Wang et al. [11] defined that the increase in battery capacity and battery size is important for the adequacy of the solar energy system. Furthermore, Abdolmaleki et al. [12] argued that a combination of hydrogen and smaller battery packs will significantly reduce system costs. In addition, efficient performance can also be achieved in this way. Farah and Anderson [13] believed that energy storage capacity costs are an important parameter for solar energy systems.

One of the important criteria for increasing the efficiency of solar energy storage investments is the use of energy storage systems in limited areas. Energy storage in limited areas is more effective and efficient than the storage system to be built in every area. Storage facilities installed in areas with high energy demand also prevent costs such as energy transfer [14]. Thus, storage activities carried out in effective and limited areas provide advantages to enterprises in terms of cost. Liang et al. [15] proposed a compressed air energy storage system to provide electricity to the green energy system with solar energy. The excess energy produced during the day is stored in this storage system.

Thus, the cost of electricity consumption is reduced thanks to the energy stored at night when the most energy is consumed. Sadeghi et al. [16] stated that thermal energy storage systems are necessary for the reuse of waste heat in places that require high energy consumption such as shopping centers. Nivolianiti et al. [17] created a microgrid method used in ships to prevent environmental degradation and fossil fuel consumption. This system will both reduce environmental degradation and provide energy efficiency with optimum system performance.

Energy optimization process with the battery system is another important criterion for increasing solar energy storage investments. The development of battery systems and the use of hybrid battery systems in appropriate structures are important in terms of energy efficiency [18]. In this way, energy will be stored and used more efficiently. [19] stated that the use of an integrated energy management system requires more than one energy source and control. They also state that energy management systems established in this way should be based on real-time prediction to be managed efficiently. Al Smadi et al. [20] developed a system using artificial intelligence technology to operate in various climatic conditions since traditional battery management systems cannot effectively optimize energy output. Kumar et al. [21] proposed an optimization method to improve energy management in storage systems. The proposed system aims to minimize costs. Boruah and Chandel [22] investigated a solar energy system with energy storage and develop a net zero energy management system.

As a result of the literature review, some important points can be reached. Firstly, solar energy projects are increasing in popularity because they reduce carbon emissions and contribute to energy independence. On the other hand, high costs and being affected by climatic differences are important barriers to the development of these projects. For these projects to be more successful, the efficiency of energy storage processes must be increased. There are many factors that affect the performance of solar energy storage investments. Improvements to be made for these factors also lead to increased costs. Therefore, to use the budget effectively and efficiently, it is appropriate for businesses to focus on more important factors. This situation increases the need for a priority analysis for performance indicators of solar energy storage investments. However, the number of studies on this subject in the literature is quite limited. In other words, this situation emerges as a significant gap in the solar energy storage investments literature. To eliminate this gap, a new model is created in this study and a priority analysis is carried out for these factors.

3. Proposed model

Determining the optimal alternative among solar energy storage systems constitutes the main problem of the study. To solve this problem of the study, the set of criteria for technical assessments of solar energy storage investments is examined. With the proposed model, the effective set of criteria for evaluating solar energy storage systems is weighted and the alternatives are then ranked based on these criteria. Multi-criteria decision making analyzes are used to solve weighting and ranking problems. While the M−SWARA analysis is carried out in weighting the criteria, the VIKOR method is applied in the ranking of solar energy storage alternatives. Experts are the data source of multi-criteria decision-making techniques used to solve problems. The fact that the data obtained from experts is a linguistic expression creates linguistic uncertainty. To analyze this type of uncertainty, fuzzy number theory of mathematics is used. The innovative fuzzy number set, in which quantum mechanics and picture fuzzy numbers are integrated with the golden ratio, is introduced in this article and applied to the methods. Additionally, an artificial intelligence model that uses demographic information is preferred so that data source experts do not have equal weight. This ensures that experts are weighted according to their expertise. In addition, since missing data may affect the analysis of the model, the missing data are integrated into the model using the recommender system with collaborative filtering method. The flowchart of the proposed model is demonstrated in Fig. 1.

The proposed model has some significant superiorities in comparison with the previously generated ones. Firstly, AI methodology is considered in this model to compute the weights of the experts. In this way, the opinions of experts with better education levels and more extensive work experience can be taken into consideration with a higher coefficient. This provides a significant advantage compared to models that evaluate the opinions of all experts with the same importance weight [23–25]. On the other hand, the use of the M−SWARA technique in the criterion weighting process also increases the quality of the model. This technique is obtained by making some improvements to the classical SWARA method. With these improvements, both the criterion weights are calculated and the causality relationship between these variables is determined. The effects of the factors used for the technical evaluation of solar energy storage investments on each other can be found. Increasing the battery capacity can also affect the provision of energy efficiency. Therefore, the M−SWARA technique is much more suitable for this study compared to other methods such as AHP and ANP [26–28]. On the other hand, the use of the VIKOR technique in the weighting process of the alternatives also increases the quality of the model. It is possible to mention some advantages of the VIKOR technique compared to TOPSIS. Although VIKOR is a distance-based method like TOPSIS, VIKOR also includes a consensus approach [29]. In other words, it allows the examination of 3 different cases such as utility, veto, and consensus with a v factor between 0 and 1 [30]. This situation also allows for more effective alternative ranking [31]. In addition to them, quantum theory is integrated with picture fuzzy rough sets. The greatest advantage of quantum theory is that it makes successful predictions [32]. Due to this advantage, this technique is used together with the fuzzy decision-making process [33]. Thus, it is possible to manage uncertainty more effectively compared to other models. Using the golden section to formulate a fuzzy set based on quantum physics also increases the quality of the proposed model [34]. The golden cut helps to define the membership and non-membership degrees of fuzzy sets more accurately by using golden ratio [35]. Similarly, the use of golden cut on the quantum fuzzy sets provides the outstanding results in the complex-decision-making process [36].

In conclusion, the two proposed multi-criteria decision-making methods and their requirements are introduced and detailed in this section. Each component of the model is presented under subheadings.

3.1. AI-based decision-making for expert prioritization

In multi-criteria decision-making methods, which is one of the problem-solving techniques, taking the average of expert evaluations in creating the decision matrix is criticized. Many different techniques are suggested in the literature to create the decision matrix. The common point of these methods is obtaining the weights of the experts and obtaining the decision matrix with the help of the weighted average. Using demographic information in determining the importance of experts raises many problems. The fact that the number of variables is large and at different scales is one of the main factors that require artificial intelligence models [37].

Experts are prioritized by applying the K-means clustering algorithm on the data set created from the demographic information of experts. After determining the optimal number of clusters (k) by the bracket method, which is performed by drawing over The Within-Cluster Sum of Squares (WCSS) value, the cluster centers (c_k), the weights of the clusters and the weights of the experts (w_{tj}) are calculated respectively using Euclidean distance (($d(x_i, x_j)$). The pseudo code of this method is shown in Table 1.

3.2. Recommender system with collaborative filtering

In the process of obtaining expert opinions, incomplete results may

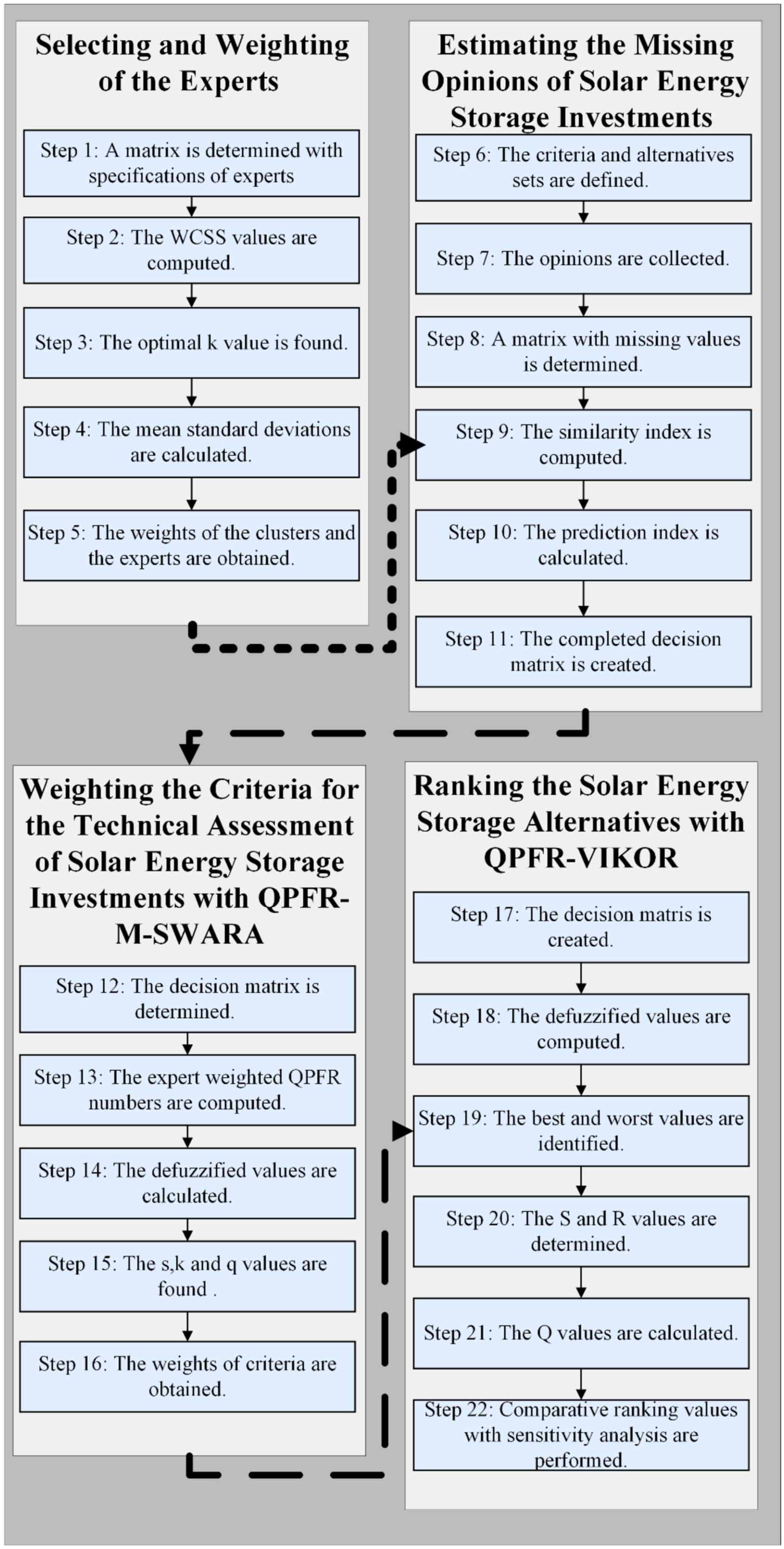

Fig. 1. Flowchart of the proposed model.

Table 1
The pseudo-code of AI-based decision making for expert prioritization.

Input: Demographic information $x_{ij}, i \in [1, m], j \in [1, n]$; where n equals number of experts, where n is number of features of experts
Output: Weight of experts $w_i, i \in [1, n]$
Begin(1)
Determine a matrix with specifications of experts
$X = [x_{ij}]_{m \times n}$
(2) Compute the WCSS values
$WCSS \leftarrow \sum_{j=1}^{k} \sum_{x_i \in C_j} d(x_i, c_j)^2; \forall k \in [1, 2, ...n]$
(3) Found the optimal k value using elbow method
(4) Assign the initial cluster centers
(5) **If** no data point changes or max iteration reaches **Then** go to (6)
Else cluster centers are updated.
$c_j \leftarrow \frac{1}{|C_j|} \sum_{x_i \in C_j} x_i; j \in [1, 2, \cdots, n]$**End If**
(6) Calculate the mean standard deviation (s_j) of each cluster
(7) Define the cluster weights (w_j)
$w_j \leftarrow |C_j| \times s_j; j \in [1, 2, \cdots, n]$(8) Compute the weights of the experts
$w_{ij} \leftarrow \frac{1}{|C_j|} \frac{w_j}{\sum_{w_j \in C_j} w_j}; j \in [1, 2, \cdots, n]$**End**

be obtained for various reasons. In the process of obtaining missing data, accessing the data source again can be difficult and may cause errors. It is possible to fill in missing data using different techniques without reaching the source. The proposed system is one of them. In this method, the missing data is computed with the similarity $(sim(u, v))$ and prediction $(p_{u,i})$ indexes of the experts. Before computing the two indexes, the experts' rating degrees $(r_{u,i}/r_{v,i})$ and average values $(\overline{r_u}, \overline{r_v})$ are found. The pseudo code of recommender system with collaborative filtering is given in Table 2 [34].

3.3. Calculating uncertainty using golden cuts based on QPFRS

There is uncertainty, or ambiguity, in linguistic evaluations. One area of mathematics called fuzzy logic is employed in the analysis of ambiguous variables. Computing that is suggested for the examination of many forms of uncertainty is fuzzy logic. Zadeh was the first to propose fuzzy logic, and it is still being worked on. This section explains fuzzy set theory definitions and how fuzzy sets based on quantum physics are formulated using golden cuts [38].

One area of physics is called quantum mechanics. With the wave function (φ), the uncertainty of a massless element's status has identified. Uncertainty is formulated with fuzzy logic. Although these two topics have not received enough attention in the literature, recent articles show how related they are. The proposed paradigm combines quantum physics with fuzzy logic. According to quantum physics, the square of φ equals the probability of the statues. These functions summarized in Equations (1) and (3) consist of a complex function within amplitude and phase angle (θ).

$$Q(|u>) = \varphi e^{j\theta} \tag{1}$$

$$|C> = \{|u_1>, |u_2>, \cdots, |u_n>\} \tag{2}$$

$$\sum_{|u> \subseteq |C>} |Q(|u>)| = 1 \tag{3}$$

Where, C is a collection of exhaustive events represented by $|u_i>$ and the degree of belief is symbolized as $|Q(|u>)| = \varphi^2$. The value of φ^2 must lie within the range of 0 to 1, and θ^2 represents the phase angle of event $|u>$. Furthermore, $|\varphi_1|^2$ serves as the degree of belief in event $|u>$, with θ representing its phase angle, which can range from 0 to 360 degrees. A traditional fuzzy number shown with Equation (4), has only a membership (μ_A). However, intuitionistic fuzzy (IFS) given by Equation (5) have non-membership (v_A) and μ_A.

$$A = \{\langle x, \mu_A(x) \rangle | x \in X\} \tag{4}$$

$$A = \{\langle x, \mu_A(x), v_A(x) \rangle | x \in X\} \tag{5}$$

Adding neutral (n_A) and refusal (h_A) degrees, picture fuzzy set (PFS) shown in Equation (6) defined.

$$A = \{\langle x, \mu_A(x), n_A(x), v_A(x), h_A(x) \rangle | x \in X\} \tag{6}$$

Basic operations for two PFSs are summarized in Equation (7).

Table 2
The pseudo-code of recommender system with collaborative filtering.

Input: Decision matrix with missing values X; $x_{ij}, i \in [1, m], j \in [1, n]$; where n and m equal dimensions of decision matrix
Output: Completed decision matrix Y;$y_{ij}, i \in [1, m], j \in [1, n]$
Begin(1)
Determine a matrix with missing values
$X = [x_{ij}]_{m \times n}$
(2) Compute the similarity index for experts
$sim(u, v) \leftarrow \sum_{i \in I} (r_{u,i} - \frac{\overline{r_u})(r_{v,i} - \overline{r_v})}{\sqrt{\sum_{i \in I} (r_{u,i} - \overline{r_u})^2} \sqrt{\sum_{i \in I} (r_{v,i} - \overline{r_v})^2}}$(3) **If** any missing values is not in data, **Then** go to (4)
Else Calculate the prediction index
$p_{u,i} = \frac{\sum_{j \in S} sim(u, v) r_{u,j}}{\sum_{j \in S} |sim(u, v)|} i \in [1, m]; \forall u$**End If**
(4) Create the completed decision matrix
$Y = [y_{ij}]_{m \times n}$
End

$$A \subseteq B \text{if } \mu_A(x) \le \mu_B(x) \text{and} n_A(x) \le n_B(x) \text{and} v_A(x) \ge v_B(x), \forall x \in X)$$
$$A = B \text{if} A \subseteq B \text{and} B \subseteq A$$
$$A \cup B = \{(x, max(\mu_A(x), \mu_B(x)), min(n_A(x), n_B(x)), min(v_A(x), v_B(x)))|x \in X\}$$
$$A \cap B = \{(x, min(\mu_A(x), \mu_B(x)), min(n_A(x), n_B(x)), max(v_A(x), v_B(x)))|x \in X\}$$
$$coA = \overline{A} = \{(x, v_A(x), n_A(x), \mu_A(x))|x \in X\} \tag{7}$$

The rough number is determined with lower ($\underline{Apr}(C_i)$)-upper ($\overline{Apr}(C_i)$) approximation and rough boundary intervals ($Bnd(C_i)$). By Equation (8), the relevant of C_i are determined.

$$\begin{aligned}
\underline{Apr}(C_i) &= \cup\left\{ Y \in \frac{X}{R(Y)} \le C_i \right\} \\
\overline{Apr}(C_i) &= \cup\left\{ Y \in \frac{X}{R(Y)} \ge C_i \right\} \\
Bnd(C_i) &= \cup\left\{ Y \in \frac{X}{R(Y)} \ne C_i \right\}
\end{aligned} \tag{8}$$

Lower ($\underline{Lim}(C_i)$), upper ($\overline{Lim}(C_i)$) limits and the rough number ($RN(C_i)$) of C_i are defined using Equation (9).

$$\begin{aligned}
\underline{Lim}(C_i) &= \sqrt[N_L]{\prod_{i=1}^{N_L} Y \in \underline{Apr}(C_i)} \\
\underline{Lim}(C_i) &= \sqrt[N_L]{\prod_{i=1}^{N_L} Y \in \underline{Apr}(C_i)} \\
\overline{Lim}(C_i) &= \sqrt[N_U]{\prod_{i=1}^{N_U} Y \in \overline{Apr}(C_i)} \\
RN(C_i) &= \left[\underline{Lim}(C_i), \overline{Lim}(C_i) \right]
\end{aligned} \tag{9}$$

Where, numbers of objects for $\underline{Apr}(C_i)$ and $\overline{Apr}(C_i)$ are N_L and N_U. QPFRS are created from PFS using the golden cuts. The set in Equation (10) has membership ($C_{i\mu_A}$), neutral (C_{in_A}), non-membership (C_{iv_A}) and refusal (C_{ih_A}) degrees.

$$|C_A> = \left\{ \begin{array}{c} \langle u, (\left[\underline{Lim}(C_{i\mu_A}), \overline{Lim}(C_{i\mu_A})\right](u), \left[\underline{Lim}(C_{in_A}), \overline{Lim}(C_{in_A})\right](u), \\ \left[\underline{Lim}(C_{iv_A}), \overline{Lim}(C_{iv_A})\right](u), \left[\underline{Lim}(C_{ih_A}), \overline{Lim}(C_{ih_A})\right](u)) | u \in 2^{|C_A>} \end{array} \right\} \tag{10}$$

The definitions of PFRS for lower limits are identified with Equation (11).

$$\begin{aligned}
\underline{Lim}(C_{i\mu_A}) &= \frac{1}{N_{L\mu_A}} \sum_{i=1}^{N_{L\mu_A}} Y \in \underline{Apr}(C_{i\mu_A}) \\
\underline{Lim}(C_{in_A}) &= \frac{1}{N_{Ln_A}} \sum_{i=1}^{N_{Ln_A}} Y \in \underline{Apr}(C_{in_A}) \\
\underline{Lim}(C_{iv_A}) &= \frac{1}{N_{Lv_A}} \sum_{i=1}^{N_{Lv_A}} Y \in \underline{Apr}(C_{iv_A}) \\
\underline{Lim}(C_{ih_A}) &= \frac{1}{N_{L\pi_A}} \sum_{i=1}^{N_{L\pi_A}} Y \in \underline{Apr}(C_{ih_A})
\end{aligned} \tag{11}$$

The components of PFRS for upper limit are identified with Equation (12).

$$\begin{aligned}
\overline{Lim}(C_{i\mu_A}) &= \frac{1}{N_{U\mu_A}} \sum_{i=1}^{N_{U\mu_A}} Y \in \overline{Apr}(C_{i\mu_A}) \\
\overline{Lim}(C_{in_A}) &= \frac{1}{N_{Un_A}} \sum_{i=1}^{N_{Un_A}} Y \in \overline{Apr}(C_{in_A}) \\
\overline{Lim}(C_{iv_A}) &= \frac{1}{N_{Uv_A}} \sum_{i=1}^{N_{Uv_A}} Y \in \overline{Apr}(C_{iv_A}) \\
\overline{Lim}(C_{ih_A}) &= \frac{1}{N_{U\pi_A}} \sum_{i=1}^{N_{U\pi_A}} Y \in \overline{Apr}(C_{ih_A})
\end{aligned} \tag{12}$$

Where $N_{L\mu_A}, N_{Ln_A}, N_{Lv_A}, N_{Lh_A}$ are the number of elements in $\underline{Apr}(C_{i\mu_A})$, $\underline{Apr}(C_{in_A})$, $\underline{Apr}(C_{iv_A})$, $\underline{Apr}(C_{ih_A})$ respectively while $N_{U\mu_A}, N_{Un_A}, N_{Uv_A}, N_{Uh_A}$ are defined for $\overline{Apr}(C_{i\mu_A}), \overline{Apr}(C_{in_A}), \overline{Apr}(C_{iv_A}), \overline{Apr}(C_{ih_A})$. The components of PFRS for lower approximation are identified with Equation (13).

$$\begin{aligned}
\underline{Apr}(C_{i\mu_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{i\mu_A} \right\} \\
\underline{Apr}(C_{in_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{in_A} \right\} \\
\underline{Apr}(C_{iv_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{iv_A} \right\} \\
\underline{Apr}(C_{ih_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{ih_A} \right\}
\end{aligned} \tag{13}$$

The components of PFRS for upper approximation are identified with Equation (14).

$$\begin{aligned}
\overline{Apr}(C_{i\mu_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{i\mu_A} \right\} \\
\overline{Apr}(C_{in_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{in_A} \right\} \\
\overline{Apr}(C_{iv_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{iv_A} \right\} \\
\overline{Apr}(C_{ih_A}) &= \cup\left\{ Y \in \frac{X}{\widetilde{R}(Y)} \le C_{ih_A} \right\}
\end{aligned} \tag{14}$$

Where $\widetilde{C_i} = (C_{i\mu_A}, C_{in_A}, C_{iv_A}, C_{ih_A})$ and $\widetilde{R}$ is the collection of $\{\widetilde{C_1}, \widetilde{C_2}, \cdots, \widetilde{C_n}\}$. $C_{i\mu_A}, C_{in_A}, C_{iv_A}, C_{ih_A}$ are the picture fuzzy sets of class $\widetilde{C_i}$. Equations (15) and (16) give information about formula of QPFRS.

$$C = \left[C_\mu.e^{j2\pi.\alpha}, C_n.e^{j2\pi.\gamma}, C_v.e^{j2\pi.\beta}, C_h.e^{j2\pi.T}\right] \tag{15}$$

$$\varphi^2 = |C_\mu(|u_i>)| \tag{16}$$

The amplitudes of quantum membership, neutral, non-membership, and refusal degrees, denoted by C_μ, C_n, C_v and C_h respectively, are represented in terms of phase angles α, γ, β, T. The amplitude of the membership function C_μ of quantum fuzzy sets is defined by φ^2. Golden ratio (G) is added to the QPFRS, then, the terms in Equations (17) and (18) are defined.

$$C_n = \frac{C_\mu}{G} \tag{17}$$

$$C_h = \frac{C_v}{G} \tag{18}$$

G equals $(1 + \sqrt{5})/2$. The phase angle of the membership degree for the probability of event $|u_i>$ in the realm of QPFS is illustrated by α in Equations (19) − (21).

$$\alpha = |C_\mu(|u_i>)| \tag{19}$$

$$\gamma = \frac{\alpha}{G} \tag{20}$$

$$T = \frac{\beta}{G} \tag{21}$$

Basic operations are shared in Equations (22)-(25). λ is any positive value.

$$\lambda^{*}\widetilde{A}_c = \left\{ \begin{array}{l} \left[\underline{Lim}\left(C_{\mu_{\tilde{A}}}\right)\lambda, \overline{Lim}\left(C_{\mu_{\tilde{A}}}\right)\lambda\right] e^{j2\pi.\left[\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)\lambda, \left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)\lambda\right]}, \left[\underline{Lim}\left(C_{n_{\tilde{A}}}\right)\lambda, \overline{Lim}\left(C_{n_{\tilde{A}}}\right)\lambda\right] e^{j2\pi.\left[\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)\lambda, \left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)\lambda\right]}, \\ \left[\underline{Lim}\left(C_{v_{\tilde{A}}}\right)\lambda, \overline{Lim}\left(C_{v_{\tilde{A}}}\right)\lambda\right] e^{j2\pi.\left[\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)\lambda, \left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)\lambda\right]}, \left[\underline{Lim}\left(C_{h_{\tilde{A}}}\right)\lambda, \overline{Lim}\left(C_{h_{\tilde{A}}}\right)\lambda\right] e^{j2\pi.\left[\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)\lambda, \left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)\lambda\right]} \end{array} \right\} \tag{22}$$

$$\widetilde{A}_c^{\ \lambda} = \left\{ \begin{array}{l} \left[\underline{Lim}\left(C_{\mu_{\tilde{A}}}\right)^{\lambda}, \overline{Lim}\left(C_{\mu_{\tilde{A}}}\right)^{\lambda}\right] e^{j2\pi.\left[\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)^{\lambda}, \left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \left[\underline{Lim}\left(C_{n_{\tilde{A}}}\right)^{\lambda}, \overline{Lim}\left(C_{n_{\tilde{A}}}\right)^{\lambda}\right] e^{j2\pi.\left[\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)^{\lambda}, \left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \\ \left[\underline{Lim}\left(C_{v_{\tilde{A}}}\right)^{\lambda}, \overline{Lim}\left(C_{v_{\tilde{A}}}\right)^{\lambda}\right] e^{j2\pi.\left[\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)^{\lambda}, \left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \left[\underline{Lim}\left(C_{h_{\tilde{A}}}\right)^{\lambda}, \overline{Lim}\left(C_{h_{\tilde{A}}}\right)^{\lambda}\right] e^{j2\pi.\left[\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)^{\lambda}, \left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)^{\lambda}\right]}, \end{array} \right\} \tag{23}$$

$$\widetilde{A}_c \cup \widetilde{B}_c = \left\{ \begin{array}{l} \left[min\left(\underline{Lim}\left(C_{\mu_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\underline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{\mu_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\underline{\alpha}_{\tilde{B}}}{2\pi}\right)}\right), max\left(\overline{Lim}\left(C_{\mu_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{\mu_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{B}}}{2\pi}\right)}\right)\right], \\ \left[min\left(\underline{Lim}\left(C_{n_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\underline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{n_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\underline{\gamma}_{\tilde{B}}}{2\pi}\right)}\right), max\left(\overline{Lim}\left(C_{n_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{n_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{B}}}{2\pi}\right)}\right)\right], \\ \left[min\left(\underline{Lim}\left(C_{v_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\underline{\beta}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{v_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\underline{\beta}_{\tilde{B}}}{2\pi}\right)}\right), max\left(\overline{Lim}\left(C_{v_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{v_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{B}}}{2\pi}\right)}\right)\right], \\ \left[min\left(\underline{Lim}\left(C_{h_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\underline{T}_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{h_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\underline{T}_{\tilde{B}}}{2\pi}\right)}\right), max\left(\overline{Lim}\left(C_{h_{\tilde{A}}}\right)e^{j2\pi.\left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{h_{\tilde{B}}}\right)e^{j2\pi.\left(\frac{\overline{T}_{\tilde{B}}}{2\pi}\right)}\right)\right] \end{array} \right\} \tag{24}$$

$$\widetilde{A}_c \cap \widetilde{B}_c = \left\{ \begin{array}{l} \left[max\left(\underline{Lim}\left(C_{\mu_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{\mu_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\alpha_{\tilde{B}}}{2\pi}\right)} \right), min\left(\overline{Lim}\left(C_{\mu_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{\mu_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\overline{\alpha}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[max\left(\underline{Lim}\left(C_{n_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{n_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\gamma_{\tilde{B}}}{2\pi}\right)} \right), min\left(\overline{Lim}\left(C_{n_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{n_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\overline{\gamma}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[max\left(\underline{Lim}\left(C_{v_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{v_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\beta_{\tilde{B}}}{2\pi}\right)} \right), min\left(\overline{Lim}\left(C_{v_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{v_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\overline{\beta}_{\tilde{B}}}{2\pi}\right)} \right) \right], \\ \left[max\left(\underline{Lim}\left(C_{h_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{T_{\tilde{A}}}{2\pi}\right)}, \underline{Lim}\left(C_{h_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{T_{\tilde{B}}}{2\pi}\right)} \right), min\left(\overline{Lim}\left(C_{h_{\tilde{A}}}\right) e^{j2\pi.\left(\frac{\overline{T}_{\tilde{A}}}{2\pi}\right)}, \overline{Lim}\left(C_{h_{\tilde{B}}}\right) e^{j2\pi.\left(\frac{\overline{T}_{\tilde{B}}}{2\pi}\right)} \right) \right] \end{array} \right\} \quad (25)$$

3.4. M−SWARA with QPFRS

When weighing the criteria, the stepwise weight assessment ratio analysis method, or SWARA method, is referred to by its acronym. Its purpose is to identify a reasonable dispute resolution procedure. The significant ratio for the criterion evaluation and the experts' hierarchical priorities serves as the foundation for this approach. This paper proposes an extension of SWARA called multi-SWARA to precisely find the weights and associated degrees of the criterion. The pseudo code of M−SWARA with QPFRS is presented in Table 3 [39].

3.5. VIKOR with QPFRS

The VIKOR approach is a MCDM process that determines a compromise outcome in an attempt to pinpoint the solution that is most similar to ideal. A compromise solution is a consensus reached by mutual concessions. Conventional methods often fail to minimize gaps in multi-criteria decision-making situations and rank alternatives as well because expert judgments of each alternative's criteria are intrinsically subjective [40]. When choices need to be made, the VIKOR method is employed to determine which set of options is the greatest fit and to provide a compromise that addresses the opposing criteria, ultimately leading to a conclusion. The pseudo code of VIKOR with QPFRS is presented in Table 4.

Table 3
The pseudo-code of M−SWARA with QPFRS.

Input: Criteria values C; $c_{ij}, i \in [1, n], j \in [1, n]$; where n equals number of criteria; weights of experts W; $w_i i \in [1, k]$; where k equals number of experts
Output: Weights of Criteria;$w_i, i \in [1, n]$
Begin(1)
Determine a matrix
$C = [c_{ij}]_{n \times n}$
(2) Compute the expert weighted QPFR numbers
$C \leftarrow w_k$ xC
(3) Calculate the defuzzified values

$$Defc_i \leftarrow \frac{\left(\begin{array}{l} \underline{Lim}(C_{\mu_i}) - \underline{Lim}(C_{n_i}) + \underline{Lim}(C_{\mu_i}).\left(\underline{Lim}(C_{v_i}) - \underline{Lim}(C_{h_i})\right) + \left(\frac{\alpha_{ij}}{2\pi}\right) - \left(\frac{\gamma_{ij}}{2\pi}\right) + \left(\frac{\alpha_{ij}}{2\pi}\right).\left(\left(\frac{\beta_{ij}}{2\pi}\right) - \left(\frac{T_{ij}}{2\pi}\right)\right) + \\ \overline{Lim}(C_{\mu_i}) - \overline{Lim}(C_{n_i}) + \overline{Lim}(C_{\mu_i}).\left(\overline{Lim}(C_{v_i}) - \overline{Lim}(C_{h_i})\right) + \left(\frac{\overline{\alpha}_{ij}}{2\pi}\right) - \left(\frac{\overline{\gamma}_{ij}}{2\pi}\right) + \left(\frac{\overline{\alpha}_{ij}}{2\pi}\right).\left(\left(\frac{\overline{\beta}_{ij}}{2\pi}\right) - \left(\frac{\overline{T}_{ij}}{2\pi}\right)\right) \end{array} \right)}{2}$$

(4) Found the comparative importance rate (s), coefficient value (k), and recalculated weight (q)
If j = 1 **Then**
$k_j = 1 q_j = 1$**Else**
$K_j \leftarrow s_J + 1$
$q_j \leftarrow q_{j-1} / k_j$
End If
(5) Calculate the weights of criteria
$w_j \leftarrow \frac{q_j}{\sum_{k=1}^{n} q_k}$(6) Compute the stable values with the large t values
End

Table 4
The pseudo-code of VIKOR with QPFRS.

Input: Opinions X; $x_{ij}, i \in [1,m], j \in [1,n]$; where n equals number of criteria and m means number of alternatives; weights of criteria W;$w_i i \in [1,n]$
Output: Rank of Alternative;$A_i, i \in [1,m]$
Begin(1)
Determine a decision matrix
$X = [x_{ij}]_{m\times n}$
(2) Compute the defuzzified values
(3) Identify the best ($\widetilde{f}_j^*$) and worst ($\widetilde{f}_j^-$) values for each criterion
$\widetilde{f}_j^* \leftarrow max_i \widetilde{x}_{ij} \widetilde{f}_j^- \leftarrow min_i \widetilde{x}_{ij}, j = 1,2,\cdots,n$(4) Determine the mean group utility ($\widetilde{S}_i$) and maximal regret ($\widetilde{R}_i$)
$\widetilde{S}_i \leftarrow \sum_{i=1}^{n} \widetilde{w}_j \frac{\left(\left|\widetilde{f}_j^* - \widetilde{x}_{ij}\right|\right)}{\left(\left|\widetilde{f}_j^* - \widetilde{f}_j^-\right|\right)} \widetilde{R}_i \leftarrow max_j \left[\widetilde{w}_j \frac{\left(\left|\widetilde{f}_j^* - \widetilde{x}_{ij}\right|\right)}{\left(\left|\widetilde{f}_j^* - \widetilde{f}_j^-\right|\right)}\right] i = 1,2,\cdots,m$(5) Calculate the final ranking of alternatives for v values between 0 and 1
$\widetilde{Q}_i \leftarrow v\left(\widetilde{S}_i - \widetilde{S}^*\right)/\left(\widetilde{S}^- - \widetilde{S}^*\right) + (1-v)\left(\widetilde{R}_i - \widetilde{R}^*\right)/\left(\widetilde{R}^- - \widetilde{R}^*\right)$(6) Ranking the alternatives
$A_i, i \in [1,m] \leftarrow$ rank the alternatives based on the order of Q values
End

Table 5
Demographic information of the experts who accepted the invitation.

Experts	Education	Experience	Salary (USD)	Age
Expert1	Master	12	2200	44
Expert2	Bachelor	14	2200	40
Expert3	Bachelor	16	2400	45
Expert4	Master	14	2200	42
Expert5	PhD	15	2350	43
Expert6	PhD	18	2400	46

4. Analysis results

The results of the study conducted to determine the most optimal alternative for solar energy storage investments are presented as subsections within this section, respectively. In the first part, obtaining the experts' weights is reported, and in the second part, the completion of missing data is reported. In the third section, the criteria in solar energy investments are weighted. In the last part, the most optimal solar energy storage alternative is determined.

4.1. Selecting and weighting of the experts

The data required for analysis is obtained from experts. In the process of selecting the experts, some criteria are taken into consideration. Experts with at least ten years of experience in solar energy storage systems are determined. These experts consist of both senior managers of companies in the market and professors who have scientific publications on the subject. In this context, a total of 6 different experts are selected. 4 of these experts are senior managers and 2 are professors. In the vast majority of similar studies in the literature, analysis is carried out with 3–6 experts [41–43]. Considering this issue, it is quite appropriate to carry out analysis with 6 different experts in this study. In this scope, a meeting invitation is made via e-mail. Online and face-to-face interviews are held with experts who accept the invitation. Demographic information of the six experts who accepted the invitation is given in Table 5.

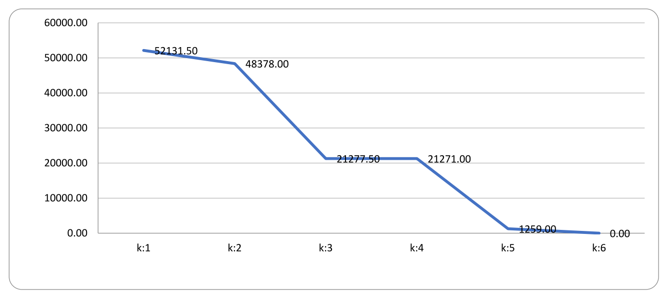

Fig. 2. The graph for Elbow method.

As can be seen in Table 5, the age range of experts is between 40 and 46 years. Additionally, two of the six experts have doctoral level education. The data set in Table 5 is used to prioritize experts. For the artificial intelligence-based expert prioritization method, the optimal number of clusters must first be determined. For this, WCSS values are calculated for different k values from 1 to 6. The results are shown in Table 6.

The elbow method is used to determine the optimal number of clusters for the K-means cluster algorithm. The graph drawn for this is visualized in Fig. 2.

According to the Elbow method, the point where the WCSS value does not decrease despite the increase in the number of clusters is the point where the k value is three. In other words, the optimal number of clusters is determined as 3. Once the optimal number of clusters is

Table 6
The WCSS values.

K:1		K:2		K:3		K:4		K:5		K:6	
Cluster 1	**VALUE**	**Cluster 1**	**VALUE**	**Cluster 1**	**VALUE**	**Cluster 1**	**VALUE**	**Cluster 1**	**VALUE**	**Cluster 1**	**VALUE**
EXPERT1	8411.25	EXPERT1	4451.67	EXPERT1	10010.25	EXPERT1	0	EXPERT1	0	EXPERT1	0
EXPERT2	8415.58	EXPERT4	4449.00	EXPERT6	10010.25	**Cluster 2**	**VALUE**	**Cluster 2**	**VALUE**	**Cluster 2**	**VALUE**
EXPERT3	11741.25	EXPERT6	17793.33	**Cluster 2**	**VALUE**	EXPERT2	0	EXPERT2	0	EXPERT2	0
EXPERT4	8405.25	**Cluster 2**	**VALUE**	EXPERT2	1.25	**Cluster 3**	**VALUE**	**Cluster 3**	**VALUE**	**Cluster 3**	**VALUE**
EXPERT5	3403.92	EXPERT2	13619.67	EXPERT4	1.25	EXPERT3	627.25	EXPERT3	0	EXPERT3	0
EXPERT6	11754.25	EXPERT3	6951.33	**Cluster 3**	**VALUE**	EXPERT5	627.25	**Cluster 4**	**VALUE**	**Cluster 4**	**VALUE**
		EXPERT5	1113.00	EXPERT3	627.25	**Cluster 4**	**VALUE**	EXPERT4	0	EXPERT4	0
				EXPERT5	627.25	EXPERT4	10008.25	**Cluster 5**	**VALUE**	**Cluster 5**	**VALUE**
						EXPERT6	10008.25	EXPERT5	629.50	EXPERT5	0
								EXPERT6	629.50	**Cluster 6**	**VALUE**
Total	52131.50	**Total**	48378.00	**Total**	21277.50	**Total**	21271.00	**Total**	1259.00	EXPERT6	0

Table 7
Iteration results for clustering.

Iteration (Expert 1 is in First Cluster; Expert 2 is in Second; Expert 3 is in Third)				
Initial Cluster Centers				
Experts	**First**	**Second**	**Third**	**Membership**
Expert1	0.00	4.58	200.04	1
Expert2	4.58	0.00	200.07	2
Expert3	200.04	200.07	0.00	3
Expert4	2.83	2.24	200.03	2
Expert5	150.04	150.05	50.09	3
Expert6	200.10	200.14	3.00	3
Average of Data Points				
Expert1	0.00	3.64	183.39	1
Expert2	4.58	1.12	183.41	2
Expert3	200.04	200.05	16.73	3
Expert4	2.83	1.12	183.37	2
Expert5	150.04	150.02	33.41	3
Expert6	200.10	200.11	16.82	3

Table 8
The Clusters' Statistics.

Code of Center	N	Edu	Exp.	$	Year	Mean of Standard Deviations	W
C1	1	0.00	0.00	0.00	0.00	0.00	0.00
C2	2	0.50	0.00	0.00	1.00	0.38	0.75
C3	3	0.94	1.25	23.57	1.25	6.75	2.26

determined, the clustering algorithm is applied for three clusters. First, the initial cluster centers of the clusters are determined. Then, with iterations, cluster centers and cluster memberships of experts are updated. Iteration results are illustrated in Table 7.

According to the clustering analysis result, the first expert is alone in the first cluster. The second cluster includes expert 2 and expert 4. The other three experts are determined as members of the third cluster. After defining the cluster memberships of the experts, the cluster weights and the weights of the experts are calculated. For this, the mean standard deviation of each cluster is obtained. The mean standard deviations of the clusters are shared in Table 8.

When Table 8 is examined, it is seen that the weight of the first cluster is 0, the weight of the second cluster is 0.75 and the weight of the last cluster is 2.26. Using the cluster weights, the experts' weights are calculated. The importance levels are displayed in Table 9.

Of the six experts, expert 3, expert 5 and expert 6 are determined to be the most important and featured experts in the study. According to the results of Table 9, expert 1 is removed from the analysis process due to its weight being zero. In other words, that Expert2, Expert3, Expert4, Expert5 and Expert6 have the priorities for analysis. So, the opinions of these five experts except Expert1 are collected for analysis.

Table 9
The weight of the experts with pareto principle.

Experts	Weights	Normalized weights with pareto principle
Expert1	0.00	0.00
Expert2	0.02	0.10
Expert3	0.32	0.27
Expert4	0.02	0.10
Expert5	0.32	0.27
Expert6	0.32	0.27

Table 10
Criteria set for the technical assessment of solar energy storage investments.

Criteria	Codes
Energy efficiency in the storage process	CR1
Battery capacity	CR2
Energy storage facilities in limited space	CR3
Energy optimization with battery management system	CR4

Table 11
Solar energy storage alternatives.

Alternatives	Codes
Storage with battery	A1
Heat capturing	A2
Pumped hydro for mechanic energy	A3
Hydrogen-based storage	A4

Table 12
Scales and QPFN values.

For Criteria	For Alternatives	Score	Pos	QPFNs
N	W	1	0.40	$\begin{bmatrix}\sqrt{.16}\,e^{j2\pi..4}, \\ \sqrt{.10}\,e^{j2\pi..25}, \\ \sqrt{.46}\,e^{j2\pi..22}, \\ \sqrt{.28}\,e^{j2\pi..13}\end{bmatrix}$
S	P	2	0.45	$\begin{bmatrix}\sqrt{.20}\,e^{j2\pi..45}, \\ \sqrt{.13}\,e^{j2\pi..28}, \\ \sqrt{.42}\,e^{j2\pi..17}, \\ \sqrt{.25}\,e^{j2\pi..10}\end{bmatrix}$
M	F	3	0.50	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi..50}, \\ \sqrt{.15}\,e^{j2\pi..31}, \\ \sqrt{.37}\,e^{j2\pi..12}, \\ \sqrt{.23}\,e^{j2\pi..07}\end{bmatrix}$
H	G	4	0.55	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55}, \\ \sqrt{.19}\,e^{j2\pi..34}, \\ \sqrt{.32}\,e^{j2\pi..07}, \\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$
VH	B	5	0.60	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi..6}, \\ \sqrt{.22}\,e^{j2\pi..37}, \\ \sqrt{.26}\,e^{j2\pi..02}, \\ \sqrt{.16}\,e^{j2\pi..01}\end{bmatrix}$

4.2. Estimating the missing opinions of solar energy storage investments

The criteria selected for the technical assessment of solar energy storage investments are coded for analysis in Table 10.

The criteria selected for analysis are approved by experts. Energy efficiency in the storage process refers to the minimization of energy loss in the process. Low energy loss has a positive impact on the increasing efficiency. On the other hand, energy efficiency allows the costs of storage processes to be reduced, which makes solar energy systems more economical. With the help of this issue, it can be more possible to attract the attention of the investors for these projects. The second criterion in this framework is battery capacity. Higher battery capacity increases the amount of energy to be stored. The biggest disadvantage of solar energy projects is the energy outage that occurs due to climatic differences. To combat this problem more effectively, it is important to increase the battery capacity. In this context, more energy can be stored, and the energy supply can be ensured to be continuous. In other words, sudden

Table 13
Experts' evaluations for the criteria.

	Expert2	Expert3	Expert4	Expert5	Expert6
CR1- CR2	n/a	5	5	5	4
CR1- CR3	3	4	4	4	n/a
CR1- CR4	4	4	n/a	4	4
CR2- CR1	4	3	5	3	4
CR2- CR3	4	4	5	3	3
CR2- CR4	5	4	5	3	4
CR3- CR1	4	3	5	3	5
CR3- CR2	n/a	2	4	4	5
CR3- CR4	4	2	4	4	4
CR4- CR1	3	2	4	2	3
CR4- CR2	3	3	4	4	2
CR4- CR3	3	3	5	5	3

Table 14
Experts' evaluations for alternatives.

	Expert2	Expert3	Expert4	Expert5	Expert6
CR1- A1	5	5	4	n/a	4
CR1- A2	5	4	4	4	5
CR1- A3	4	5	4	4	5
CR1- A4	n/a	5	3	4	5
CR2- A1	4	4	4	n/a	5
CR2- A2	3	4	5	4	4
CR2- A3	n/a	3	4	4	4
CR2- A4	5	3	3	5	3
CR3- A1	4	4	3	4	3
CR3- A2	3	n/a	4	3	4
CR3- A3	3	5	4	3	3
CR3- A4	4	n/a	4	2	4
CR4- A1	n/a	5	4	3	5
CR4- A2	4	5	5	4	4
CR4- A3	4	4	5	5	5
CR4- A4	5	4	5	5	4

Table 15
Similarity index matrix for the criteria.

	Expert2	Expert3	Expert4	Expert5	Expert6
Expert 2	1.00	0.25	0.50	−0.25	0.52
Expert 3	0.25	1.00	0.55	0.32	−0.05
Expert 4	0.50	0.55	1.00	0.04	0.17
Expert 5	−0.25	0.32	0.04	1.00	−0.01
Expert 5	0.52	−0.05	0.17	−0.01	1.00

changes in the amount of solar energy production can be balanced with the increase in battery capacities. Establishing energy storage facilities in a limited area allows the storage system to be customized. This is especially critical for urban areas where space constraints exist. By establishing these facilities, the space-consuming problem of energy storage facilities can be minimized. Thus, it is possible to carry out solar energy projects more effectively in regions with limited residential areas. The last criterion included for analysis is energy optimization with the battery management system. Battery management and efficiency will be ensured with the system to be installed. Solar energy storage alternatives are displayed in Table 11.

One of the alternatives identified for investment is storage with battery. In this context, electrical energy is stored as chemical energy. When necessary, it is also possible to convert it back into electrical energy. This produced energy is stored in batteries. In cases where demand is high, this energy is made available for use. Heat capturing is the second alternative. In this context, it is possible to collect and reuse the heat produced in various processes. In other words, this collected heat is converted into energy by using it. Heat capture increases energy efficiency and provides less energy consumption. Pumped hydro for mechanic energy is another way of solar energy storage alternative. This system allows water to be pumped between two reservoirs and stored. In cases of high energy demand, this stored water is used to produce electrical energy with the help of turbines. Hydrogen-based storage is also recommended as other alternatives. In this method, hydrogen gas can be stored in tanks or in liquefied form. The stored hydrogen can be used to produce energy when needed. Evaluations are obtained from experts to calculate the weights of the criteria and determine the most appropriate alternative. The linguistic expressions used in the evaluation of experts and the QPFN values corresponding to the linguistic expressions are given in Table 12.

Experts evaluate criteria together using linguistic expressions.

Table 16
Similarity index matrix for the alternatives.

	Expert2	Expert3	Expert4	Expert5	Expert6
Expert 2	1.00	−0.23	−0.17	0.48	0.12
Expert 3	−0.23	1.00	0.10	−0.41	0.28
Expert 4	−0.17	0.10	1.00	0.14	0.24
Expert 5	0.48	−0.41	0.14	1.00	0.03
Expert 5	0.12	0.28	0.24	0.03	1.00

Table 17
Iterative completion of missing expression for the criteria.

	Expert2	Expert3	Expert4	Expert5	Expert6
CR1- CR2	4 (First)	5	5	5	4
CR1- CR3	3	4	4	4	3 (First)
CR1- CR4	4	4	4 (First)	4	4
CR2- CR1	4	3	5	3	4
CR2- CR3	4	4	5	3	3
CR2- CR4	5	4	5	3	4
CR3- CR1	4	3	5	3	5
CR3- CR2	5 (First)	2	4	4	5
CR3- CR4	4	2	4	4	4
CR4- CR1	3	2	4	2	3
CR4- CR2	3	3	4	4	2
CR4- CR3	3	3	5	5	3

Table 18
Iterative completion of missing expression for the alternatives.

	Expert2	Expert3	Expert4	Expert5	Expert6
CR1- A1	5	5	4	5 (First)	4
CR1- A2	5	4	4	4	5
CR1- A3	4	5	4	4	5
CR1- A4	4 (First)	5	3	4	5
CR2- A1	4	4	4	4 (First)	5
CR2- A2	3	4	5	4	4
CR2- A3	4 (First)	3	4	4	4
CR2- A4	5	3	3	5	3
CR3- A1	4	4	3	4	3
CR3- A2	3	4 (First)	4	3	4
CR3- A3	3	5	4	3	3
CR3- A4	4	4 (First)	4	2	4
CR4- A1	3 (First)	5	4	3	5
CR4- A2	4	5	5	4	4
CR4- A3	4	4	5	5	5
CR4- A4	5	4	5	5	4

Table 19
Completed linguistic opinions of experts for the criteria.

	CR1	CR2	CR3	CR4
EXPERT 2				
CR1		H	M	H
CR2	H		H	VH
CR3	H	VH		H
CR4	M	M	M	
EXPERT3				
CR1		VH	H	H
CR2	M		H	H
CR3	M	S		S
CR4	S	M	M	
EXPERT4				
CR1		VH	H	H
CR2	VH		VH	VH
CR3	VH	H		H
CR4	H	H	VH	
EXPERT5				
CR1		VH	H	H
CR2	M		M	M
CR3	M	H		H
CR4	S	H	VH	
EXPERT6				
CR1		H	M	H
CR2	H		M	H
CR3	VH	VH		H
CR4	M	S	M	

Table 20
QPFN for the relationship matrix.

	CR1	CR2	CR3	CR4
EXPERT2				
CR1		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR2	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$
CR3	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR4	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	
EXPERT3				
CR1		$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR2	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR3	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.20}\,e^{j2\pi.45},\\ \sqrt{.13}\,e^{j2\pi.28},\\ \sqrt{.42}\,e^{j2\pi.17},\\ \sqrt{.25}\,e^{j2\pi.10}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.20}\,e^{j2\pi.45},\\ \sqrt{.13}\,e^{j2\pi.28},\\ \sqrt{.42}\,e^{j2\pi.17},\\ \sqrt{.25}\,e^{j2\pi.10}\end{bmatrix}$
CR4	$\begin{bmatrix}\sqrt{.20}\,e^{j2\pi.45},\\ \sqrt{.13}\,e^{j2\pi.28},\\ \sqrt{.42}\,e^{j2\pi.17},\\ \sqrt{.25}\,e^{j2\pi.10}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	
EXPERT4				
CR1		$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR2	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$
CR3	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR4	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	
EXPERT5				
CR1		$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR2	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$
CR3	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR4	$\begin{bmatrix}\sqrt{.20}\,e^{j2\pi.45},\\ \sqrt{.13}\,e^{j2\pi.28},\\ \sqrt{.42}\,e^{j2\pi.17},\\ \sqrt{.25}\,e^{j2\pi.10}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	
EXPERT6				
CR1		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$

Table 20 (*continued*)

	CR1	CR2	CR3	CR4
EXPERT2				
CR2	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR3	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi.6},\\ \sqrt{.22}\,e^{j2\pi.37},\\ \sqrt{.26}\,e^{j2\pi.02},\\ \sqrt{.16}\,e^{j2\pi.01}\end{bmatrix}$		$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi.55},\\ \sqrt{.19}\,e^{j2\pi.34},\\ \sqrt{.32}\,e^{j2\pi.07},\\ \sqrt{.19}\,e^{j2\pi.04}\end{bmatrix}$
CR4	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.20}\,e^{j2\pi.45},\\ \sqrt{.13}\,e^{j2\pi.28},\\ \sqrt{.42}\,e^{j2\pi.17},\\ \sqrt{.25}\,e^{j2\pi.10}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi.50},\\ \sqrt{.15}\,e^{j2\pi.31},\\ \sqrt{.37}\,e^{j2\pi.12},\\ \sqrt{.23}\,e^{j2\pi.07}\end{bmatrix}$	

Experts' evaluations for the criteria are summarized in Table 13.

Similarly, experts evaluate alternatives based on criteria and the results are reported in Table 14.

In Table 13 and Table 14, missing data are expressed as n/a. The values of missing data are estimated with the recommender system method. In this method, first, the similarity ratios of experts are calculated. The similarity index of the experts according to Table 13 are represented in Table 15.

The similarity index of the experts according to Table 14 for alternatives are represented in Table 16.

Using the similarity index in Table 15, missing data in the criterion matrix is completed as a result of the iterations. The results are shown in Table 17.

Similarly, with the help of the values in Table 16, the missing data of the alternative matrix is completed. The final decision matrix used in the analysis is displayed in Table 18.

4.3. *Weighting the criteria for the technical assessment of solar energy storage investments with QPFR-M−SWARA*

The values in Table 17 are used in calculating the weights of technical assessments of solar energy storage investments. The linguistic equivalents of the values completed as a result of the iteration are shared in Table 19.

QPFN of expert opinions are determined. The values for the relationship matrix are shown in Table 20.

Using the experts' weights in Table 9, the weighted average of the five experts' evaluations is calculated. Expert weighted QPFRS for the direct relation is illustrated in Table 21.

The defuzzified values of the values in Table 21 are calculated. Defuzzified values are summarized in Table 22.

Next, the defuzzied values are normalized. The normalized relation matrix calculated as a result of the normalization process are given in Table 23.

In the next step, the values of S, k q and w are obtained with the values of the normalized relationship matrix. The results of these values are detailed in Table 24.

The relationship matrix is created with the W values calculated for each criterion in Table 24. The average of this matrix is considered as the threshold value. Values above this value are defined as affecting the row criterion and affecting the column criterion. The relationship matrix and impact directions are summarized in Table 25.

According to Table 25, CR1 affects CR2. Similarly, CR2 affects CR4, CR3 affects CR2, and finally CR4 affects CR3. The power of the relationship matrix is taken to calculate the weights of the criteria selected for the technical assessment of solar energy storage investments. Thus, the values are made stationary. The result of the 2 t + 1th power of the relationship matrix is displayed in Table 26, with the t value being a very large value.

As can be seen in Table 26, the values in the columns of the stable matrix are the same. That is, the values of the matrix are stationary.

Table 21
The direct relation matrix.

	CR1	CR2	CR3	CR4
CR1		$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$
CR2	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$		$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$
CR3	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$		$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$
CR4	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil.05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil.01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil.05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil.03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil.01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil.01,.01\rfloor}$	

Table 22
Defuzzified values.

	CR1	CR2	CR3	CR4
CR1	0.000	0.068	0.060	0.062
CR2	0.062	0.000	0.062	0.064
CR3	0.068	0.069	0.000	0.063
CR4	0.056	0.061	0.066	0.000

Table 23
Normalized relation matrix.

	CR1	CR2	CR3	CR4
CR1	0.000	0.356	0.318	0.326
CR2	0.330	0.000	0.330	0.341
CR3	0.340	0.344	0.000	0.315
CR4	0.303	0.334	0.362	0.000

According to the result of the stable matrix, the most important criterion for the technical assessment of solar energy storage investments is CR2 with the largest value of the matrix, 0.282. The second most important criterion among the selected criteria is CR3. According to these results, battery capacity and limited area energy storage facilities are important issues in the technical assessment of solar energy investments. High battery capacity or the condition of the facilities may make it possible to evaluate investments.

4.4. Ranking the solar energy storage alternatives with QPFR-VIKOR

The values in Table 18 are used in ranking of the solar energy storage alternatives in Table 11. The linguistic equivalents of the values completed as a result of the iteration are shared in Table 27.

QPFN of linguistic expressions using Table 12 are given in Table 28 for the decision matrix.

The weighted averages of these values are calculated using expert weights. The expert weights here are the values in Table 9. The expert-weighted decision matrix obtained as a result of the calculation is summarized in Table 29.

The defuzzified values of the values in Table 29 are computed. The results are given in Table 30.

In the next step, S and R values are calculated. With these values, Q values are calculated for different values of v. Q values calculated for the value of V between 0 and 1 with 0.1 increments are shown in Table 31.

After the two assumptions of the VIKOR method are tested, the results are interpreted. As can be seen in Table 31, in the case of consensus where v is equal to 0.5, the best alternative is Pumped hydro for mechanic energy. The ranking of the alternatives is similar in cases where V is greater than or less than 0.5. In all three cases, the most suitable solar energy storage alternative is Pumped hydro for mechanic energy.

4.5. Comparative ranking values with sensitivity analysis

In the literature on multi-criteria decision making, sensitivity analysis addresses the change in the ranking of alternatives in terms of criteria weights. In other words, it examines the extent to which small changes in criteria values can be effective in selecting the optimal solution. In addition, sensitivity analysis is also preferred to analyze the consistency of the change in ranking depending on the change in criteria

Table 25
The relationship matrix and impact directions.

	CR1	CR2	CR3	CR4	Impact directions
CR1		0.430	0.246	0.324	CR1 → CR2
CR2	0.244		0.324	0.431	CR2 → CR4
CR3	0.323	0.432		0.245	CR3 → CR2
CR4	0.247	0.322	0.430		CR4 → CR3

Table 26
Stable Matrix.

	CR1	CR2	CR3	CR4
CR1	0.213	0.213	0.213	0.213
CR2	0.282	0.282	0.282	0.282
CR3	0.252	0.252	0.252	0.252
CR4	0.253	0.253	0.253	0.253

Table 24
S, k, q and w for each criterion.

CR1	**S**	**k**	**q**	**W**	**CR2**	**S**	**k**	**q**	**W**
CR2	0.356	1.000	1.000	0.430	CR4	0.341	1.000	1.000	0.431
CR4	0.326	1.326	0.754	0.324	CR1	0.330	1.330	0.752	0.324
CR3	0.318	1.318	0.572	0.246	CR3	0.330	1.330	0.566	0.244
CR3	**S**	**k**	**q**	**W**	**CR4**	**S**	**k**	**q**	**W**
CR2	0.344	1.000	1.000	0.432	CR3	0.362	1.000	1.000	0.430
CR1	0.340	1.340	0.746	0.323	CR2	0.334	1.334	0.749	0.322
CR4	0.315	1.315	0.567	0.245	CR1	0.303	1.303	0.575	0.247

Table 27
Completed linguistic opinions of experts for the alternatives.

	CR1	CR2	CR3	CR4
EXPERT2				
A1	B	G	G	F
A2	B	F	F	G
A3	G	G	F	G
A4	G	B	G	B
EXPERT3				
A1	B	G	G	B
A2	G	G	G	B
A3	B	F	B	G
A4	B	F	G	G
EXPERT4				
A1	G	G	F	G
A2	G	B	G	B
A3	G	G	G	B
A4	F	F	G	B
EXPERT5				
A1	B	G	G	F
A2	G	G	F	G
A3	G	G	F	B
A4	G	B	P	B
EXPERT6				
A1	G	B	F	B
A2	B	G	G	G
A3	B	G	F	B
A4	B	F	G	G

values. In sensitivity analysis, the effect of the weight values of the criteria on the results in different scenarios is analyzed. Thus, the effect of possible situation changes on the ranking of alternatives can be examined. Moreover, sensitivity analysis provides feedback for improving decision support systems and models. This leads to more accurate and reliable decisions. Analyzes are repeated using different weights and comparison method, taking into consideration the consensus status in VIKOR. Thus, the sensitivity in the ranking of solar energy storage alternatives is tested. TOPSIS is recommended for the comparison method in the study. As a result, the ranking results of the TOPSIS method and the VIKOR method in consensus calculated with four different weights are illustrated with the graph in Table 32.

It is seen that the ranking results are similar in all calculated cases. This situation gives information about the consistency of the results. This explains that the results of the decision model do not change if the weights of the criteria used are changed. Hence, it can be understood that the proposed model provides reliable findings. Pumped hydro for mechanic energy is found as the best alternative for solar energy storage. According to the sensitivity analysis results, this alternative is determined as the most suitable investment option.

5. Discussion

This study analyses the criteria that are effective in increasing solar energy storage investments. The development of solar energy storage investments is effective in increasing the use and performance of renewable energy. Renewable energy can be used effectively thanks to the sufficient solar energy storage capacity. It is possible to reuse the energy produced in excess of the consumed amounts when needed by increasing the storage capacity. At the same time, the development of battery capacity is important in storing intermittent energy sources such as solar energy and reusing them when needed. Battery capacity is also very critical for energy continuity. Sufficient battery capacity allows for more storage of the energy produced by solar panels. Some interruptions in energy supply may occur, especially during hours when the sun is low. This situation leads to significant customer dissatisfaction. Increasing battery capacity is of critical importance to effectively manage this problem. On the other hand, sufficient battery capacity also helps

Table 28
QPFN for the decision matrix.

	CR1	CR2	CR3	CR4
EXPERT2				
A1	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$
A2	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
EXPERT3				
A1	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
A2	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$
EXPERT4				
A1	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$
A2	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
EXPERT5				
A1	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$
A2	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.20}e^{j2\pi.45},\\ \sqrt{.13}e^{j2\pi.28},\\ \sqrt{.42}e^{j2\pi.17},\\ \sqrt{.25}e^{j2\pi.10}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$
EXPERT6				
A1	$\begin{bmatrix}\sqrt{.30}e^{j2\pi.55},\\ \sqrt{.19}e^{j2\pi.34},\\ \sqrt{.32}e^{j2\pi.07},\\ \sqrt{.19}e^{j2\pi.04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}e^{j2\pi.50},\\ \sqrt{.15}e^{j2\pi.31},\\ \sqrt{.37}e^{j2\pi.12},\\ \sqrt{.23}e^{j2\pi.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}e^{j2\pi.6},\\ \sqrt{.22}e^{j2\pi.37},\\ \sqrt{.26}e^{j2\pi.02},\\ \sqrt{.16}e^{j2\pi.01}\end{bmatrix}$

(*continued on next page*)

Table 28 (*continued*)

EXPERT2				
	CR1	**CR2**	**CR3**	**CR4**
A2	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi..6},\\ \sqrt{.22}\,e^{j2\pi..37},\\ \sqrt{.26}\,e^{j2\pi..02},\\ \sqrt{.16}\,e^{j2\pi..01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55},\\ \sqrt{.19}\,e^{j2\pi..34},\\ \sqrt{.32}\,e^{j2\pi..07},\\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55},\\ \sqrt{.19}\,e^{j2\pi..34},\\ \sqrt{.32}\,e^{j2\pi..07},\\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55},\\ \sqrt{.19}\,e^{j2\pi..34},\\ \sqrt{.32}\,e^{j2\pi..07},\\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi..6},\\ \sqrt{.22}\,e^{j2\pi..37},\\ \sqrt{.26}\,e^{j2\pi..02},\\ \sqrt{.16}\,e^{j2\pi..01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55},\\ \sqrt{.19}\,e^{j2\pi..34},\\ \sqrt{.32}\,e^{j2\pi..07},\\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi..50},\\ \sqrt{.15}\,e^{j2\pi..31},\\ \sqrt{.37}\,e^{j2\pi..12},\\ \sqrt{.23}\,e^{j2\pi..07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi..6},\\ \sqrt{.22}\,e^{j2\pi..37},\\ \sqrt{.26}\,e^{j2\pi..02},\\ \sqrt{.16}\,e^{j2\pi..01}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{.36}\,e^{j2\pi..6},\\ \sqrt{.22}\,e^{j2\pi..37},\\ \sqrt{.26}\,e^{j2\pi..02},\\ \sqrt{.16}\,e^{j2\pi..01}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.25}\,e^{j2\pi..50},\\ \sqrt{.15}\,e^{j2\pi..31},\\ \sqrt{.37}\,e^{j2\pi..12},\\ \sqrt{.23}\,e^{j2\pi..07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55},\\ \sqrt{.19}\,e^{j2\pi..34},\\ \sqrt{.32}\,e^{j2\pi..07},\\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$	$\begin{bmatrix}\sqrt{.30}\,e^{j2\pi..55},\\ \sqrt{.19}\,e^{j2\pi..34},\\ \sqrt{.32}\,e^{j2\pi..07},\\ \sqrt{.19}\,e^{j2\pi..04}\end{bmatrix}$

optimize solar energy production. In other words, efficiency in operational processes can increase thanks to more stored solar energy. This situation allows for significant cost reductions. As a result of this issue, long-term profitability of projects can be increased. Increasing battery capacity also increases the use of renewable energy. The biggest disadvantage of renewable energy projects is that they are affected by climatic conditions. Thanks to the increased capacity, this problem can be minimized. This situation provides a significant competitive advantage to renewable energy projects. With the help of this condition, investors give more importance to these projects. It is possible to find many studies advocating this view in the literature. Gomes et al. [44] proposed the use of lithium-ion batteries for the storage of solar energy. They stated that the energy stored in this way provides cost advantage. Tong et al. [45] demonstrated that small and widely used batteries will be more efficient in terms of solar energy storage. It is also identified that in addition to battery capacity, hybrid storage systems will also be effective for solar energy storage systems.

In addition, the use of battery systems for energy optimization provides advantages to businesses in terms of efficiency and costs. In this way, it is possible to store energy efficiently and at low cost. Additionally, it prevents unnecessary energy consumption by regulating energy consumption. This contributes significantly to extending the life of the batteries. In this way, the cost effectiveness and reliability of solar energy systems increases. As a result, the return-on-investment period is shortened. On the other hand, energy optimization reduces unnecessary expenses. This allows energy to be used at lower costs. Thanks to the battery management system, it is possible to ensure efficiency and energy continuity. Using this system, the charging and discharging processes of batteries can be optimized. Owing to this condition, energy continuity is provided. This situation allows customer expectations to be met to a significant extent. The battery management system also contributes to extending the life of batteries. With this system, the temperature, voltage and current levels of battery cells can be continuously monitored. This situation also supports the reduction of issues that will damage batteries, such as overheating. On the other hand, another advantage of this system is that it reduces energy losses. Thus, it is possible to increase the effectiveness of projects. Similarly, with the help of the battery management system, energy demand can be predicted in advance. By taking this data into account, the capacity of the batteries can be adjusted. In this way, energy supply and demand are balanced more efficiently. This situation also allows the efficiency of the process to be increased significantly. There are studies defending this view in the literature. For instance, Sipra et al. [46] reached a conclusion that a hybrid battery management system will be suitable for solar energy storage. On the other side, Poti et al. [47] underlined that that battery management increases energy reliability and availability.

Table 30
The defuzzified decision values.

	CR1	CR2	CR3	CR4
A1	0.068	0.068	0.060	0.066
A2	0.068	0.060	0.060	0.068
A3	0.068	0.062	0.066	0.068
A4	0.066	0.066	0.063	0.070

Table 31
S, R and Q values.

	S	R	Q1	Q2	Q3	Q4
A1	0.515	0.253	0.662	0.643	0.623	0.603
A2	0.698	0.282	1.000	1.000	1.000	1.000
A3	0.343	0.189	0.000	0.000	0.000	0.000
A4	0.403	0.213	0.244	0.235	0.227	0.219
	Q5	**Q6**	**Q7**	**Q8**	**Q9**	**Q 1**
A1	0.584	0.564	0.544	0.525	0.505	0.485
A2	1.000	1.000	1.000	1.000	1.000	1.000
A3	0.000	0.000	0.000	0.000	0.000	0.000
A4	0.210	0.202	0.194	0.186	0.177	0.169

6. Conclusion

This study makes evaluation for the technical assessment of solar energy storage investments. Within this framework, a new four-stage model is generated by considering different decision-making techniques and fuzzy sets. The first stage is related to the prioritizing the experts with AI-based decision-making method. Secondly, the missing evaluations of solar energy storage investments are estimated with expert recommender system. In the following part, the criteria for the technical assessment of solar energy storage investments are weighted by QPFRS adopted M−SWARA. The final stage consists of ranking the solar energy storage alternatives with QPFR-VIKOR. It is identified that battery capacity is the most critical factor for the technical assessment of solar energy storage investments because it has the greatest weight (0.282). Energy optimization with battery management system (weight:

Table 29
Expert weighted QPFRS for the decision matrix.

	CR1	CR2	CR3	CR4
A1	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil .05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil .01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$
A2	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil .05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil .01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil .05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil .01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$
A3	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil .05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil .01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$
A4	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$	$\lceil\sqrt{.03},\sqrt{.08}\rfloor e^{j2\pi.\lceil .05,.15\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .03,.09\rfloor},\lceil\sqrt{.04},\sqrt{.10}\rfloor e^{j2\pi.\lceil .01,.03\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .01,.02\rfloor}$	$\lceil\sqrt{.03},\sqrt{.10}\rfloor e^{j2\pi.\lceil .05,.16\rfloor},\lceil\sqrt{.02},\sqrt{.06}\rfloor e^{j2\pi.\lceil .03,.10\rfloor},\lceil\sqrt{.04},\sqrt{.09}\rfloor e^{j2\pi.\lceil .01,.02\rfloor},\lceil\sqrt{.02},\sqrt{.05}\rfloor e^{j2\pi.\lceil .01,.01\rfloor}$

Table 32
Comparative Ranking Results with Sensitivity Analysis.

Alternatives	Ranking Results with TOPSIS				Ranking Results with VIKOR			
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
A1	3	3	3	3	3	3	3	3
A2	4	4	4	4	4	4	4	4
A3	1	1	1	1	1	1	1	1
A4	2	2	2	2	2	2	2	2

0.253) and energy storage facilities in limited space (weight: 0.252) are also critical in this respect. However, energy efficiency in the storage process has the lowest weight (0.213). On the other hand, pumped hydro for mechanic energy is found as the most significant solar energy storage alternative.

The main contribution of this study is the generation of an original decision-making model. In this model, decision matrix by the help of AI. This situation gives an opportunity to calculate the significance weights of the experts. Therefore, the analysis results can be more reliable and coherent. Similarly, M−SWARA methodology is considered to compute the weights of the criteria. With the help of this situation, causality relationship between the determinants can be defined. In addition to them, the integration of quantum theory with picture fuzzy rough sets has a positive influence on managing uncertainties more effectively. In summary, this proposed model has some superiorities by comparing with the previously generated models. This model can be considered for defining strategic investment decisions especially for energy projects. Energy investments usually involve complex and multi-dimensional decision processes. The main reason for this is that these projects usually require the evaluation of more than one alternative. These processes require the consideration of many factors such as financial, technical, environmental, social and political. Therefore, this model is quite useful and practical for making comprehensive, balanced and sustainable decisions in energy investments.

However, the main limitation of this study is that a general evaluation is performed with respect to the solar energy storage investments. Thus, a country specific analysis can be conducted in the future studies. For example, making an analysis for developing countries gives an opportunity to generate significant investment strategies for these countries. This situation can be helpful for these countries to have sustainable economic growth. In addition to this issue, the recommended methodology has also some limitations. With respect to the ranking of the alternatives, VIKOR and TOPSIS approaches are taken into consideration. Nevertheless, these techniques are also criticized by the scholars. Because of this condition, the improved versions of these techniques can be generated for future research direction. Furthermore, to weight the experts, limited determinants are considered. For future studies, more variables can be taken into consideration to reach this objective. With the help of this situation, it is possible to calculate the importance weights of these experts more precisely.

CRediT authorship contribution statement

Gang Kou: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Hasan Dinçer:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Serhat Yüksel:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Serkan Eti:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Merve Acar:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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