

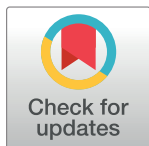
RESEARCH ARTICLE

Entropy-based financial asset pricing: Evidence from Pakistan

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Abstract

Entropy is an alternative measure to calculate the risk, simplify the portfolios and equity risk premium. It has higher explanatory power than capital asset price model (CAPM) beta. The comparison of Entropy and CAPM beta provide in depth analysis about the explanatory power of the model that in turn help investor to make right investment decisions that minimizes risk. In this context, this study aims to compare Shannon and Rennyi Entropies with the CAPM beta for measuring the risk. Ordinary Least square approach has been utilized using a dataset of 67 enterprises registered in Pakistan Stock exchange. The comparative analysis of CAPM beta and entropy has been carried out with the R^2 parameters. The result indicates that entropy has more explanatory power as compare to CAPM beta's explanatory power, and this turns out to be the best option to evaluate the risk performances. The result implies that an investor should make the best investment decision by choosing an enterprise that provide with good returns at minimum risk based on entropy technique.

1. Introduction

This ingenious measure of risk in the probability hypothesis may be calculated successfully for resource-evaluating models. To illustrate the susceptibility of an irregular variable, the entropy of that variable is used to illustrate the risk of stock and portfolio returns. Entropy uses measures of probability to estimate the danger of safety, whereas the standard way to calculating hazard makes use of CAPM beta. It has been claimed that earnings are assumed to be set and consistently appropriated by [1]. Entropy was used as an alternative to difference in hazard estimate in the [2] since it is a fraction of vulnerability in the probability hypothesis. After that, [3] demonstrated that the entropy is used in domains where probabilities are still up in the air. [3] a approach was presented by [4] for determining which facts of the entropy measure are most important to resource returns. Examples of monetary applications of entropy include portfolio advancement [5, 6] and decision assessing [7]. The illustrative force of the entropy percentage of hazard is significantly bigger than the conventional estimations when depicting

Abbreviations: CAPM, Capital Asset Price Model; OLS, Ordinary Least Square; PSX, Pakistan Stock Exchange; CDE, Credit Default Exchange; CRM, Credit Risk Market.

the arrival of an enormous example of stocks and portfolios. Although entropy decreases due to widening in much the same way as standard deviation, it also captures an intentional beta-like risk of single protections or ineffective portfolios, as shown by their investigation. Entropy has 1.5 times the logical force of the beta of the capital resource valuing model (CAPM) beta in very large portfolios [6]. Furthermore, describing any market portfolio isn't necessary for entropy estimate. Because Beta relies solely on previous results, it does not take into account any new information that might alter future returns. The proportion of Beta and the cost of value alter as more return data is accumulated over time. As a result, entropy is being used as an exam-specific optional metric. Using the usual least square (OLS) relapse setting, we found that the informative power is substantially greater in entropy than conventional hazard assessments. The common least squares (OLS) method is used to evaluate the ambiguous borders in a straight relapse model. For each combination of illustrative factors, OLS determines the maximum number of squares that can be distinguished between what is known about the observed ward variable (benefits associated with the observed variable) and what is predicted by that variable's direct capacity. For the purposes of this investigation, the informational index for the Pakistan Stock Exchange will be examined for entropy and CAPM beta. The research used histogram-based thickness capacities and the danger estimations' informative power on a large example of Pakistan's stock exchange (PSX). In total, there are 67 organizations in the database that span the period from 2005 to 2019. CAPM beta and entropy have been studied up close using R2 bounds. Using PSX data, we may compare the entropy and CAPM beta measurements and make a determination of their relationship. With the use of three control parameters.

The remaining of this research is organized as follows: Section 1 includes the introduction. Section 2 present literature review. Methodology and data used in this research is given in section 3. The result and discussion are given in section 4. While, Section 5 conclude this research.

2. Literature review

Entropy has been the subject of extensive research during the last many years. However, there aren't a lot of studies on entropy in finance. Examining the writing hole is done through the composition of many articles. Entropy, according to [6], produces better results than the Capital Consent Value Model (CAPM) beta. Using entropy, [8] found that it has a higher illustrative value than CAPM beta when it comes to assessing risk. In the person of [9] Tsallis' total entropy reveals less unexpected behavior than the other three danger metrics when it comes to the two respectabilities of fit and the variety of rewards associated with risk. [10] found that the Paris List was more volatile from 2000 to 2012 than any other major record. In their findings, [11] found a link between shared data and efficient danger and between explicit danger and contingent entropy. Using [12]'s results, they demonstrate that entropy may serve as a proxy for portfolio board vulnerability on the Portuguese stock exchange, demonstrating the validity of entropy as a proxy for portfolio board vulnerability. Greetings, [13] their findings reveal that while oil and capriciousness information is scarce before to the crisis, it becomes more significant during the crisis when oil and unsteadiness information are more widely available. According to [14], the entropy displays a skillful anticipating capacity to forecast banking and financial crises. It is argued that entropy provides a natural speculative foundation for dealing with such speculative and partial information. Quantile backslides allow us to examine the effect of entropy on VAR's quantiles in greater detail. In the person of [15]. According to their findings, yager's entropy yields a larger monetary value for extension than Shannon entropy and the minimax uniqueness model, and as a result, the market may be

more effectively changed by reallocating assets. There are two ways in which information may be exchanged between the two markets, however the Credit Default Exchange (CDE) overpowers the Credit Risk Market (CRM). My name is [15] Liberal bundling and a crisis-like structure are shown to be less consistent for the majority of market structure given the external and internal occurrences of mental maltreatment, political and monetary crises for Pakistan. According to [16], a more modest portfolio turnover occurs when all of the variables associated with change, skewness, and entropy are tied to the actual job. Aiming towards optimum portfolio affirmation, [17] developed mean-entropy-skewness models. As a result of its independence from the balanced probability stream, entropy is preserved as a measure of risk. On the topic of entropy in finance, [18] conducted a study that focused on portfolio assurance and asset appraisal. They also highlighted how entropy affects the strategy as well as how it differs from other methods. Everything about entropy that's useful to experts is covered in [19] survey. They provide a range of entropy measurements. In the case of [20] For the capital asset assessment model and the Fama French three and five factor models, their results reveal that the entropy piece is indisputably compared to the GLS R2 estimate [21]. When a quantile relapse increases, the CAPM beta danger turns into a varied positive. These results suggest that the entropy assessor performs somewhat better than the next assessor. [21, 22] in the US saw a fall in productivity across all four corporate sectors when the Coronavirus devastated everything in its path. His findings expand on two previously unexplored locations. First and foremost, the market's overall productivity has declined due to the Coronavirus, and bitcoin's efficacy has declined across a wide range of industries, leading its founder to dub the cryptocurrency a "refuge resource" because of this. In addition, [23] Dubai's record returns are closely linked to the model's entropy, as seen by these findings. Entropy theory and Markowitz portfolio modelling go hand in hand in this groundbreaking demonstration of how expected return and risk go hand in hand. [24] argues that entropy may cause problems in a situation without the need for rigid ideas that might influence the results. In two or three assessments, entropy as a deficit is separated and changed to demonstrate and reachability of entropy as an elective degree of scattering [20]. Qualitative and entropy-based surveys are used by [23] to describe dispersals and probable outcomes, although they believe there is no unlimited link between these activities and mentioning spreading. To sum up, the research presented here indicates that entropy may be used to accurately estimate the threat. The Capital Resource Estimating Model is a common balancing model in monetary writing. [25] explained that in high-risk protections, CAPMS, the financial backer isn't competent to rely on the CAPM since the CAPMS isn't exact and the regular speed of profit from speculation isn't normal. According to [26], the multi-hazard model and GARCH models, which are more precise than the CAPM model in calculating the rate of profit from an interest in the offers, have distinct repercussions. From 1973 to 1978, [27] analyzed the data on the 220 protected persons, including the 26-year-old. In order to deal with the orderly market risk factors and the administration of risk factors, they use two simple models. [15] found that CAPM produced more accurate results in short-term speculation when paired with long-term investments. CAPM short-run results on the KSE should be highlighted by the financial supporter, according to the authors. There are no lingering consequences of this evaluation on Pakistani protections trading from [28]. This has varying effects on all three types of beta. The results of this study are consistent with other research conducted in Pakistan Stock business sectors, although throughout a variety of time periods and sample sizes. As a result, theories and propositions asserted against the CAPM were found to be invalid, according to [28]. As [29] reveals, for rare connections and over short periods of time, CAPM provides cautious results. With just 28 out of 360 viewpoints supporting CAPM, and 332 rejecting the paradigm in this institutional packaging effort, CAPM is exonerated. Hey there, [30] Using a sample of 45 companies, the researchers

discovered that the beta gains for 37 of the companies were fundamental, indicating that beta remains an extremely high level of risk in a developing economy like Pakistan. This finding shows that [31, 32]-style CAPM is one of the most important frameworks for evaluating the cost of significant worth capital in developing economies like Pakistan.

3. Methodology

We use three risk methods named CAPM Beta and two Entropies (Shannon and Rennyi). CAPM beta is a traditional way of measuring safety risks. Four decades later, the CAPM is still widely used in applications such as determining a firm's cost of capital and monitoring the performance of managed portfolios. The CAPM's appeal is that it provides powerful and intuitively satisfying results.

3.1 Data sample

Our observed evaluation is based on Pakistan Stock Exchange markets <https://www.psx.com.pk/>, for which the KSE100 index is taken as a market standard. An index includes all the major companies of the whole country Pakistan. We selected 67 stocks that are comprised of the KSE 100 index. The risk-free rate is the 3 months T bills rate and is risk free because they are fully backed by the government. Three control variables are also included which are turnover ratio, leverages, market capital/ firm size. The monthly data is obtained from the Karachi Stocks for the period from 2005 to 2019.

3.2 Variables definitions

3.2.1 Dependent variable. 3.2.1.1. *Average excess return of stock.* Also known as expected risk premium is the expected return on an asset that is higher than the risk-free rate of return. The risk premium on an asset is a sort of incentive for investors. It's a way of rewarding investors for taking on more risk in each investment than they would in a risk-free asset.

3.2.2 Independent variables. 3.2.2.1. *CAPM beta.* Beta is a percentage of an investment or portfolio's unpredictability or orderly hazard, rather than the market's overall risk to show how risk and expected return are linked in the capital resource valuation model (CAPM) (typically stocks).

3.2.2.2. *Shannon entropy.* The degree of unpredictability or uncertainty in a market or security is referred to as entropy.

3.2.2.3. *Rennyi entropy.* This entropy is same as Shannon entropy but difference in the formula, with $\alpha = 2$.

3.2.3 Control variables. 3.2.3.1. *Turnover ratio.* The asset turnover ratio compares the value of a business's assets to its sales or incomes. The asset turnover ratio is a metric that determines how well a company uses its assets to generate revenue.

3.2.3.2. *Leverages.* Leverage is the process of increasing an investment's potential return by using borrowed money, financial instruments, or borrowed capital.

3.2.3.3. *Market capital/firm size.* The total dollar market value of a company's outstanding shares of stock is referred to as market capitalization. We used the value of market capital as the value of firm size.

3.3 Research design

The Capital Asset Pricing Model (CAPM) is a mathematical model that explains how the expected return and risk of an investment are linked. It shows that the risk-free return plus a risk premium based on the security's beta equals the expected return on a security.

$$RA = RFR + [B^*(RM - RF)]$$

RA = Expected return on a security

RFR = Risk-free rate

B = Beta of the security

RM = Expected return of the market

The formula of CAPM beta is as follows.

$$\beta_i = \frac{cov(R_i - R_F, R_m - R_F)}{\sigma^2(R_m - R_F)}$$

R_i = Return of the stock

R_f = Risk-free rate

R_m = Return of the market

$R_i - R_m$ = Excess return of the stock

$R_m - R_f$ = Excess return of the market.

σ^2 = Square of Variance

β_i = Beta of security i.

3.3.1 Fuzzy entropy. The uncertainty caused by a lack of information because of failing to precisely forecast stated values is known as fuzzy entropy. The fuzzy entropy value function is as follows:

$$E[\xi] = \int_0^\infty C_r\{\xi \geq r\}dr - \int_{-\infty}^0 C_r\{\xi \leq r\}dr$$

Were,

$$Cr\{\xi \in A\} = \frac{1}{2}(sup_{x \in A}\mu(x) + 1 - sup_{x \in A}\mu(x))ue, \quad sup_{x \in A}\mu(x) = Post\{\xi \in A\}$$

In the real number system, R is a subset of all real numbers. The function is also known as the probability distribution. If at least one of the two integrals is finite, the equation is a Choquet integral. The Choquet integral is sometimes seen as a generalization of mathematically expected values when evaluating measurement theories. The system's entropy is therefore defined as:

$$H(\xi) = \sum_{i=1}^{\infty} S(Cr\{\xi = x_i\})$$

Where $S(t) = -t \ln t - (1-t) \ln(1-t)$ with the convention that

$$H(\xi) = \int_{-\infty}^{+\infty} S(Cr\{\xi = r\})dr$$

Cross entropy is defined as

$$D[\xi; \eta] = \int_{-\infty}^{+\infty} T(c_r\{\xi = x\}, c_r\{\eta = x\})dx$$

3.3.2 Tsallis relative entropy. Probability used for finite set and Tsallis relative entropy is given as:

$$T_q(P, P_0) \equiv \int_{\Sigma} P(x) \frac{[P(x)/P_0(x)]^{q-1} - 1}{q-1} dx$$

3.3.3 Shannon entropy. A probability measure's Shannon entropy on a finite set X is given by:

$$S_n(P) = - \sum_{i=1}^n p_i \ln p_i$$

$$\sum_{i=1}^n p_i = 1 \geq 0$$

When working with continuous probability distributions, a density function is evaluated at all values of the argument. The entropy of a continuous probability distribution with a density function $f(x)$ can be calculated as follows:

$$H = - \int_{-\infty}^{+\infty} f(x) \ln f(x) dx$$

So, we are using Shannon entropy, due to continuous probability distribution.

3.4 Discrete entropy function

Differential and discrete entropy functions are the two main types of entropy functions.

Suppose that the discrete random variable is represented by X^* and the probabilities are denoted by $p_i = \Pr(X^* = o_i)$, $p_i \geq 0$, and $\sum_{i=1}^n p_i = 1$. We define the general function of the distinct entropy of the variable X^* is as follows:

$$H_{\alpha}(X^*) = \frac{1}{1-\alpha} \log \left(\sum_{i=1}^n p_i^{\alpha} \right) \quad (1)$$

The entropy method is, 1 and 0, respectively. Where the base of the logarithm is $\alpha = 2$. The order of entropy determines the weight of each result; if the entropy order is low, the potential consequences are given less weight $\alpha = 2$ and $\alpha = 1$ are the maximum applied requests.

A specific case of standard entropy is entropy with a value of one. The replacement of $\alpha = 1$ in (1), on the other hand, results in zero division. Using the l'Hopital law of $\alpha = 1$ limit, it can be demonstrated that H transforms to Shannon entropy:

$$H_1(x^*) = - \sum_{i=1}^n p_i \log(p_i) \quad (2)$$

Rényi entropy is the situation where $\alpha = 2$. The formula is as follows:

$$H_2(x^*) = - \log \left(\sum_{i=1}^n p_i^2 \right) \quad (3)$$

$H_\alpha(X)$ is a function that does not grow in α , and both entropy steps are large there is zero if there are a limited number of possible consequences.

$$0 < H_2(x^*) \leq H_1(x^*) \quad (4)$$

3.4.1 Differential entropy function. Assume that X is a continuous random variable with a probability density function $f(x)$ that accepts values from $\mathbb{R}$. The definition of continuous entropy is:

$$H_\alpha(x) = \frac{1}{1-\alpha} \ln \int f(x)^\alpha dx \quad (5)$$

The bases of the logarithms in (1) and (5) differ, as can be shown. Despite the fact that entropy is reliant on the base, it can be demonstrated that the entropy value for different bases only varies by a constant coefficient. For all differential entropy functions, we utilize the natural logarithm. The formulas for the two situations ($\alpha = 1$ and $\alpha = 2$) are as follows:

$$H_1(x) = -\int f(x) \ln f(x) dx \quad (6)$$

$$H_2(x) = -\ln \int f(x)^2 dx \quad (7)$$

The difference between discrete and continuous entropy is that, unlike discrete entropy, continuous entropy can take negative values as well:

$$H_\alpha(x) \in \mathbb{R} \quad (8)$$

3.5 Entropy estimation

To estimate differential entropy, the probability density function of the return values must be computed. Let $x_1, x_2, \dots, x_n$ represent the observations of the continuous random variable X , and $H_{\alpha,n}(X)$ represent H_α 's sample-based estimation (X). The density function estimation is used to produce the entropy estimates in the plug-ins. The integral estimate of entropy, $f_n(x)$, estimates the probability density function $f(x)$ as follows:

$$H_{\alpha,n}(x) = \frac{1}{1-\alpha} \ln \int_{A_n} f_n(x)^\alpha dx \quad (9)$$

where A_n integration distance, which can eliminate insignificant values and the end of $f_n(x)$. Author suggest choosing A_n ($\min(x)$, $\max(x)$).

3.6 Histogram based density estimation method

Histogram-based density estimate is one of the most basic methods of density estimation. Let $b = (\max(x), \min(x))$ be the sample value range; divide it into k bins of equal width and label the cutting points with t_j . The width of the bin is constant: $h = \frac{b_n}{k} = t_{j+1} - t_j$. The probability density function is determined by the formula as follows:

$$f_n(x) = \frac{v_j}{nh} \quad (10)$$

A simple non-plug-in estimation formula for Shannon and Rennyi entropy may be obtained using (6), (7), (9) and (10) as well as the properties of the histogram:

$$H_{1,n}(x) = \frac{1}{n} \sum_{j=1}^k v_j \ln \left(\frac{v_j}{nh} \right) \quad (11)$$

$$H_{2,n}(x) = -\ln \sum_{j=1}^k h\left(\frac{v_j}{nh}\right)^2 \quad (12)$$

$$k_H(S_i) = e^{H_n(R_i - R_F)} \quad (13)$$

The observations are $R: R_1, R_2, \dots, R_l$, and the elements are the set of securities $S: S_1, S_2, \dots, S_l$ ($r_{i1}, r_{i2}, \dots, r_{in}$). $RM(r_{M1}, r_{M2}, \dots, r_{Mn})$ is the market return observation, while $RF(r_{F1}, r_{F2}, \dots, r_{Fn})$ is the risk free return observation, where n denotes the number of samples and l signifies the number of securities. Keep in mind that the project's main purpose is to create entropy as a new risk metric. We introduce k as a unifying feature for securities to handle the risk measure generically. Let $k(S_i)$ be the risk estimate for security i .

The k_H values are non-negative real numbers.

$$k_H \in [0, +\infty)$$

3.7. Predictive and explanatory power

We provide a basic estimate strategy, the evaluation of in-sample explanatory power, to compare efficiency risk estimation strategies.

3.7.1 In-sample. Suppose V remain a single explanatory variable by sample $u = u_1, u_2, \dots, u_l$ and U be a only target variable with sample $v = v_1, v_2, \dots, v_l$. The explanatory power of the variable U for the variable V is determined using the approach described below. The linear regression model can be used to describe the linear relationship between the two variables:

$$V = a_0 + a_1 U + \varepsilon. \quad (14)$$

Ordinary least squares (OLS) is used to estimate the model's parameters (a_0 and a_1), and the target value is calculated as $v_i = a_0 + a_1 u_i$, where a_0 and a_1 are the a_0 and a_1 estimations, respectively. One of the most often used measures of explanatory power is the R^2 (goodness of fit, or coefficient of determination) in linear regression:

$$R^2(v, u) = 1 - \frac{\sum_{i=1}^n (v_i - (a_0 + a_1 u_i))^2}{\sum_{i=1}^n (v_i - \bar{v})^2} \quad (15)$$

We're interested in seeing how well alternative risk measures represent a security's expected return, which we refer to as $g(k)$. The risk measure of the securities is the explanatory variable U , and the sample is:

$$u_k = (k(S_1), k(S_2), \dots, k(S_l)) \quad (16)$$

The predicted risk premium of the securities is the objective variable T , and the sample is:

$$v_\mu = (E[R_1 - R_F], E[R_2 - R_F], \dots, E[R_l - R_F]) \quad (17)$$

where k is the unified risk measure function, and $E[\cdot]$ is the anticipated value of the argument. The R^2 of the previously defined variables (24) and (25) is used to estimate the in-sample explanatory power (efficiency):

$$\eta(k) = R^2(v_u, u_k) \quad (18)$$

3.8 Empirical model

We constructed the model for this investigation based on the models developed by [33–36].

$$\text{Model:1 } AER = \beta_0 + \beta_1 CBETA + \beta_2 TOR + \beta_3 LEV + \beta_4 FSIZE$$

$$\text{Model:2 } AER = \beta_0 + \beta_1 SE + \beta_2 TOR + \beta_3 LEV + \beta_4 FSIZE$$

$$\text{Model:3 } AER = \beta_0 + \beta_1 RE + \beta_2 TOR + \beta_3 LEV + \beta_4 FSIZE$$

Where: AER, CBETA, SE, RE, TOR, LEV, SIZE denote average excess return, capital asset price beta, Shannon entropy, Rennyi entropy, Turnover ratio, leverages and firm size.

4. Results and discussions

We investigate whether entropy can be measured by the decrease of risk through variation. Authors make random, equal-weighted portfolios, each with a diverse number of participating securities of 67 randomly selected securities of the Pakistan Stock Exchange. Portfolio risk based on the Shannon, Rennyi and the beta of CAPM estimates, the use of the full-period risk-free. The entropy function is given by the estimate of the density function based on the histogram. We evaluated the technique using a histogram, and it proved to be the most effective in terms of predictive power, explanatory power, and simplicity. We will evaluate the risk for individual security, as well as the application of a beta, CAPM, and the entropy measure, to determine predicted risk premium may be successfully identified by risk measures. All the table shows the regression results for the linear risk in terms of risk to the monthly data. With the variation in the frequency of the observations, the results remained stable at the amount of the comparative volatility of the expected returns of the specific securities.

4.1 Diagnostic test

In this section, the correlation coefficients between the explanatory variables are seen to show the direction and degree of the link between any pair of explanatory factors and the explained variable using a correlation matrix. If the correlation coefficient is more than 0.7, there is proof of the presence of the multicollinearity problem. There is no association if the correlation coefficient is less than 0.7. Multi-collinearity is a problem [37–39].

The correlation coefficients between the variables used in this study are provided in Table 1. The correlation coefficient implies that there is no Multicollinearity problem, as seen in the table above, because there is no correlation coefficient larger than (0.70).

Table 2 shows the mean, standard deviations, minimum, and maximum values for each independent variable, as well as the mean, standard deviations, lowest, and maximum values for each dependent variable. These were the ones that were used in this study.

The Hausman test is used to identify which model (fixed or random) effect is best for analysis, with rejection meaning that the random effect model is better and failure implying that the

Table 1. Correlation matrix results.

	AER	CBETA	SE	RE	TOR	LEV	SIZE
AER	1						
CBETA	0.0295	1					
SE	0.0532	0.016	1				
RE	0.2126	-0.095	0.0027	1			
TOR	-0.085	-0.0367	0.0222	-0.0474	1		
LEV	0.0007	0.0083	-0.0056	0.0088	-0.0318	1	
SIZE	0.0252	-0.0054	0.0131	-0.0339	0.0454	-0.0231	1

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Table 2. Summary statistics.

Variable	Obs	Mean	Std. Dev.	Min	Max
AER	1005	-0.0136392	0.0558347	-0.2448323	0.1782841
CBeta	1005	0.074228	0.2869312	-1.342495	1.383375
Shannon	1005	1.10714	0.4093749	0.6215253	8.859635
Rennyi	1005	1.078416	0.2949364	0.4539983	6.080336
TOR	1005	3.449064	11.80099	-6.401594	163.4293
LEV	1005	108.5777	344.6308	0.02	5698.66
Size/ MC	1005	16.3608	1.588092	0.5784	21.505

<https://doi.org/10.1371/journal.pone.0278236.t002>

fixed effect model is best. Because all of the probability values in Table 3 are less than the 5% significance level, the fixed effect is employed.

In theory, a beta smaller than 1.0 indicates that the security is less volatile than the market, as seen in Table 4. When this stock is included in a portfolio, it reduces the portfolio's risk. A security's or an investment portfolio's sensitivity or link to market fluctuations is measured by the Beta coefficient. We can acquire a statistical measure of risk and determine the fraction of risk that can be ascribed to the market by comparing the returns of an individual security/portfolio to the returns of the general market. With a coefficient of 0.0006389, the data show that turnover has a very statistically significant negative effect on stock return. If all other factors remain constant, a one-unit increase in turnover ratio would result in a 0.06389 percent decrease in average excess return of stock. The higher the turnover ratio for enterprises listed on the Pakistan Stock Exchange, the worse the Average excess return of stock. With a coefficient of -3.98E-06, the data show that leverage has a statistically insignificant negative influence on stock performance. This finding backs with the signaling idea Increasing a company's debt ratio in its capital structure by borrowing, according to this idea, sends a favorable signal to investors that the company is borrowing money to expand its activities in profitable initiatives, resulting in increased profits. Reduce the risk of default and lower the needed return for investors. That a one-unit rise in leverage would be beneficial. If everything else remained the same, the average excess return of stock would drop by around 0.0000398 percent. The results demonstrate that the size of the firm has a statistically small effect on stock return, with a coefficient of 0.00223. If all other factors remain constant, a one-unit increase in scope of the firm would result in a 0.223 percent increase in average excess return of stock.

The Shannon entropy value is less than 1.0 in Table 5, indicating that the securities are less volatile than the market. When this stock is part of a portfolio, it reduces the risk.

With a coefficient of 0.0006305, the data show that turnover has a very statistically significant negative effect on stock return. If all other factors remain constant, a one-unit increase in turnover ratio (TURN) would result in a 0.06305 percent decrease in average excess return of stock. The higher the turnover ratio for enterprises listed on the Pakistan Stock Exchange, the worse the Average excess return of stock. With a coefficient of -3.76E-06, the data show that leverage has a statistically insignificant negative influence on stock performance. This finding backs with the signaling idea. Increasing a company's debt ratio in its capital structure by borrowing, according to this idea, sends a favorable signal to investors that the company is

Table 3. Hausman test results.

	CBETA	SE	RE
Chi2	12.20	12.02	11.84
PROBABILITY	0.0160	0.0172	0.0186

<https://doi.org/10.1371/journal.pone.0278236.t003>

Table 4. Beta fixed effect results.

AER	Coef.	Std. Err.	t	P>t	R-square
CBETA	.0479942	.0216653	2.22	0.030	0.0319
TOR	-0.0006389	0.0001748	-3.66	0	
LEV	-3.98E-06	6.38E-06	-0.62	0.533	
SIZE	2.23E-03	1.47E-03	1.52	0.13	
Constant	-0.0478576	0.0242113	-1.98	0.048	

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borrowing money to expand its activities in profitable initiatives, resulting in increased profits. Reduce the risk of default and the required return on investment for investors. That an increase in leverage of one unit would be advantageous. The average excess return on stock would fall by roughly 0.0000376 percent if everything else remained the same. The results suggest that the size of the firm has a statistically insignificant effect on stock return, with a coefficient of 0.002362. If all other factors remain constant, a one-unit increase in size of the firm would result in a 0.236 percent increase in average excess return of stock.

In Table 6 a Rennyi entropy value is less than 1.0 indicates that the securities are less volatile than the market. When this stock is included in a portfolio, it makes it less risky.

With a coefficient of 0.0005784, the data show that turnover has a very statistically significant negative effect on stock return. If all other factors remain constant, a one-unit increase in turnover ratio (TURN) would result in a 0.05784 percent decrease in average excess return of stock. The higher the turnover ratio for enterprises listed on the Pakistan Stock Exchange, the worse the Average excess return of stock. With a coefficient of -4.96E-06, the data show that leverage has a statistically insignificant negative influence on stock performance. This finding backs with the signaling idea. Increasing a company's debt ratio in its capital structure by borrowing, according to this idea, sends a favorable signal to investors that the company is borrowing money to expand its activities in profitable initiatives, resulting in increased profits. Reduce the risk of default and the required return on investment for investors. That an increase in leverage of one unit would be advantageous. The average excess return on stock would fall by roughly 0.0000496 percent if everything else remained the same. The results suggest that the firm's volume has a statistically small impact on stock return, with a coefficient of 0.002496. If all other factors remain constant, a one-unit increase in size of the firm would result in a 0.2496 percent increase in average excess return of stock.

Table 7 monthly data is displayed, as well as the results of risk premium linear regressions on risk measures. Because of the variety in observation frequency, the results are robust to the level of relative volatility of predicted returns on specific stocks. The histogram-based density-calculated Shannon entropy has the maximum explanatory power of 5.4 percent. With 4.9 percent explanatory power, other rennyi entropy measures do slightly worse. Both entropies outperform CAPM beta, which only explains 3.1 percent of the variance. Our result is consistent with the previous study of [8].

Table 5. Shannon fixed effect results.

AER	Coef.	Std. Err.	t	P>t	R-square
SE	0.1631184	.0341918	4.77	0.000	0.0547
TOR	-0.0006305	0.0001747	-3.61	0	
LEV	-3.76E-06	6.38E-06	-0.59	0.556	
SIZE	0.002362	0.0014712	1.61	0.109	
Constant	-0.0414529	0.0243693	-1.7	0.089	

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Table 6. Rennyi entropy fixed effect results.

AER	Coef.	Std. Err.	t	P>t	R square
RE	0.0401088	0.0063841	6.28	0	0.0494
TOR	-0.0005784	0.0001715	-3.37	0.001	
LEV	-4.96E-06	6.26E-06	-0.79	0.429	
SIZE	0.0024961	0.0014406	1.73	0.083	
Constant	-0.0951981	0.0249171	-3.82	0	

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Table 7. Comparison of three models with R² approach.

AER	Coef.	Std. Err.	t	P>t	Rsquare
CBETA	.0479942	.0216653	2.22	0.030	0.0319
SE	0.1631184	.0341918	4.77	0.000	0.0547
RE	0.0401088	0.0063841	6.28	0	0.0494

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5. Conclusion and suggestions

In asset pricing and financial modeling, entropy is extremely important. However, the use of the entropy technique is still debatable. Since, there are several tools and procedures for the estimation of the entropy. However, the gap is still there in the determining the value of entropy model. The comparative analysis is done on the dataset extracted from Pakistan Stock Exchange database. The experimental result is shown that the entropy (Shannon and Rennyi) is a robust risk-measurement solution and better than the beta version of the CAPM.

Although, we did not directly compare our results with those of [6] approach. Shannon entropy has the highest explanatory power that is widely used in information theory. The Histogram based density estimation function is used to calculate the entropy values. OLS regression is used to calculate the R², while R² parameter is used to compare the explanatory power of both models. The beta has 3.1%, Shannon entropy has 5.4% and Rennyi entropy has 4.9% explanatory power respectively. It demonstrates that entropy explains more than CAPM beta.

5.1. Future research work

We have utilized the control variable also, but deva approach does not include control Parameters. This research will help the investor to measure the risk efficiently. In future we will use the Kullback cross entropy, Tsallis entropy and fuzzy Entropy to improve the efficiency of risk measure.

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