

Human-computer interaction using artificial intelligence-based expert prioritization and neuro quantum fuzzy picture rough sets for identity management choices of non-fungible tokens in the Metaverse

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Abstract

Necessary improvements should be made to increase the effectiveness of non-fungible tokens on the Metaverse platform without having extra costs. For the purpose of handing this process more efficiently, there is a need to determine the most important factors for a more successful integration of non-fungible tokens into this platform. Accordingly, this study aims to determine the appropriate the identity management choices of non-fungible tokens in the Metaverse. There are three different stages in the proposed novel fuzzy decision-making model. The first stage includes prioritizing the expert choices with artificial intelligence-based decision-making methodology. Secondly, the criteria sets for managing non-fungible tokens are weighted by using Quantum picture fuzzy rough sets-based M-SWARA methodology. Finally, the identity management choices regarding non-fungible tokens in the Metaverse are ranked with Quantum picture fuzzy rough sets oriented VIKOR. The main contribution of this study is that artificial intelligence methodology is integrated to the fuzzy decision-making modelling to differentiate the experts. With the help of this situation, it can be possible to create clusters for the experts. Hence, the opinions of experts outside this group may be excluded from the scope. It has been determined that security must be ensured first to increase the use of non-fungible tokens on the Metaverse platform. Similarly, technological infrastructure must also be sufficient to achieve this objective. Moreover, biometrics for unique identification has the best ranking performance among the alternatives. Privacy with authentication plays also critical role for the effectiveness of this process.

1. Introduction

Non-fungible token is a type of crypto asset developed on blockchain technology. One of the most important features of these tokens is that they are immutable and unique. In other words, each token has its own unique digital content. This provides some benefits for increasing the efficiency of transactions to be carried out in the metaverse (Behl et al., 2024). Thanks to these tokens, it is possible to customize digital assets on the Metaverse platform. This significantly contributes to reducing the uncertainty in the process. Investors who have more confidence in the process pay more attention to this area. Thus, it is possible to increase investments. On the other hand, non-fungible tokens secure digital property rights within the metaverse. This significantly helps to increase investors' confidence in the process. Moreover, it is possible to determine the value of digital assets on the Metaverse platform by using these tokens (Lee et al., 2024). This situation supports increasing the trade volume in the process. Similarly, non-fungible tokens can be backed by smart contracts. This situation contributes to increasing the transaction volume on the Metaverse platform.

The development of non-fungible tokens is crucial to the effectiveness of the metaverse platform. First, user convenience must be ensured to increase the use of these tokens. Users want to use these tokens easily and quickly. In this context, interfaces must be designed in a user-friendly manner. In this context, it would be appropriate to provide some training to use these tokens effectively (Davies et al., 2024). Ensuring the security of the process is also very important in this context. Thanks to a secure platform, users' anxiety about using these tokens is reduced. Investors who feel safer will make more transactions on this platform. Thus, it is possible to increase the effectiveness of the metaverse platform. In this context, some actions need to be taken to ensure account confidentiality and payment security. To increase the use of non-fungible tokens on the Metaverse platform, sufficient technological infrastructure must be provided (Brickman et al., 2024). Owing to the use of advanced technologies, the blockchain system can be more successfully integrated into the metaverse platform. This practice can also significantly increase the effectiveness of the process.

To increase the effectiveness of non-fungible tokens on the Metaverse platform, necessary improvements must be made regarding these issues. However, these improvements also cause costs to increase. This situation emerges as the most important barrier in the improvement process. Therefore, making too many improvements will put businesses in a very difficult financial situation. To manage this problem effectively, improvements on more important issues must be made first. Therefore, it is necessary to identify the factors that are most important in this process. However, there are a limited number of studies focusing on this issue in the literature. This can be considered as one of the important deficiencies in the literature. In other words, a new study is needed to determine the most important factors for a more successful integration of non-fungible tokens into this platform.

Accordingly, this study aims to determine the appropriate the identity management choices of non-fungible tokens in the Metaverse. There are three different stages in the proposed novel fuzzy decision-making model. The first stage includes prioritizing the expert choices with artificial intelligence-based decision-making methodology. Secondly, the criteria sets for managing non-fungible tokens are weighted by using Quantum picture fuzzy rough sets-based M-SWARA methodology. Finally, the identity management choices regarding non-fungible tokens in the Metaverse are ranked with Quantum picture fuzzy rough sets oriented VIKOR. The main motivation of making this study is the strong need for a new fuzzy decision-making model. The main reason is that most of the existing models in the literature do not consider expert differentiation (Ecer et al., 2024; Mikhaylov et al., 2024). However, the qualifications of the experts can be different based on their demographic characteristics. Hence, a new model is needed to consider this situation to reach more appropriate results.

The main contributions of this study are demonstrated below. (i) Artificial intelligence methodology is integrated to the fuzzy decision-making modelling to differentiate the experts. With the help of this situation, it can be possible to create clusters for the experts. Hence, the opinions of experts outside this group may be excluded from the scope. This situation allows to reach more realistic analysis results. On the other side, this condition has a powerful contribution to increase methodological originality. (ii) M-SWARA methodology is preferred to weight the determinants. Owing to this issue, the causal directions among these items can be considered. The classical SWARA approach cannot create impact relation map of the criteria. Due to this situation, M-SWARA methodology is generated in the previous studies while making some improvements. As a result, the causal directions of the determinants can be identified. The influencing factors of managing non-fungible tokens may have an impact on each other. Thus, to reach more essential items, causal relationship among them should be considered. Because of this condition, M-SWARA is the most appropriate methodology in this framework.

Literature review is given in the next step. The following step consists of the details of the proposed methodology. Analysis results are explained in the fourth step. The final sections include discussion and conclusion.

2. Literature Review

NFTs, digital representations of items stored in the blockchain that cannot be exchanged with each other, feature uniqueness by utilizing smart contracts (Bonnet & Teuteberg, 2023). The development and expansion of NFT applications are directly linked to blockchain and Metaverse innovations. In this context, structural innovations in blockchain technologies are carried out on the axis of data collection, storage, sharing, interoperability, and privacy protection (Huynh-The et al., 2023). For example, Wu L. et al. (2023) developed a waste material passport as NFT, which allows cross-border trade of construction waste materials using blockchain infrastructure. With these suggestions, they aimed to ensure that waste materials have a digital and unique identity, to ensure information transparency, to increase trade efficiency, and to ensure the security of transaction records. On the other hand, Rani et al. (2023) developed an innovative blockchain-based NFT payment system for academic payment systems. In this way, an online payment model has been introduced to increase security by eliminating the lack of transparency and traceability in traditional payment methods. Ko et al. (2023) emphasize ongoing research on developing innovative NFT technologies, especially fractional NFTs, non-transferable and soul bound tokens.

Security is another issue that will increase the effectiveness of Blockchain and Non-Fungible Tokens. With the increase in its use and value, the concern that NFTs can be used to launder illegal revenues has also increased (Mosna & Soana, 2023). As a matter of fact, Rahman and Jin (2023) suggested establishing tax control to develop the Chinese NFT market and eliminate tax irregularities. They emphasized that in this way, the legality of the transactions will be ensured, as well as their security. Schlatt et al. (2023) emphasized that research on the cyber security of blockchain applications focuses on the technical infrastructure, but it is also essential to investigate the socio-technical aspects. On the other hand, since blockchain-based NFT applications provide security, they can be integrated into production processes in many different sectors. For example, Alkhader et al. (2023) found that these applications resist security attacks in their study investigating the use of blockchain and NFTs in digital production systems. Furthermore, Delgado-von-Eitzen et al. (2024) have presented a proposal for publishing and verifying academic information by the General Data Protection Regulation by taking advantage of NFTs' unique and secure structures.

Increasing the infrastructure efficiency of Non-Fungible Tokens is not enough on its own. It is a process that proceeds in parallel with the strategic establishment of business models and achieving profitability to increase innovations in this field. Thanks to NFTs, new business models can be developed focusing on digital ownership, digital assets that can be moved and combined, and decentralized communities (Li & Chen, 2023). At this point, as stated by Cohan and Paschen (2023), marketing professionals need to be guided in persuading consumers to increase their interest in NFTs. Belk et al. (2022) contributed to the field by theorizing new forms of ownership so that NFT and web 3.0 applications can become widespread, especially in the art world, and artists can earn money. Similarly, Mancuso et al. (2023) presented a framework for companies to create added value by supporting their investments in Metaverse and Web 3.0 technologies with new business models. On the other hand, Lee et al. (2024) evaluated the effectiveness of the opportunities provided by the platforms in their research on the effect of NFT use on customer loyalty. Accordingly, they emphasized that customer loyalty increases thanks to the opportunities provided by NFT platforms and that it is essential to design a user-centered experience.

As mentioned above, another criterion that increases the effectiveness of NFTs is user experience. Ali et al. (2023) stated that these tools will be adopted more as the interfaces of NFT platforms are designed to be user-friendly. For instance, Lee (2023) investigated the factors affecting users' adoption of NFTs. Accordingly, usability is among the technical factors that affect users. Moreover, the research conducted by Sun (2023) revealed that one of the leading reasons for choosing NFT platforms is ease of use. However, Wu C. et al. (2023) emphasized in their research on end NFT users that usability is one of the biggest challenges of current applications. Finally, Meyns and Dalipi (2022) investigated the concerns of end NFT users about the platform in their research. According to research findings, users are concerned about access to NFT platforms by special invitation, verification processes, restricting account access, or removing their own NFTs. As a result, eliminating access barriers and usage concerns of end users is one of the crucial criteria for increasing NFT effectiveness.

The diversity and effectiveness of Web 3.0 applications is increasing day by day. It is seen that these technologies change the way different sectors do business. Blockchain-based NFT applications are among the most frequently emphasized and researched investments in recent years. In this respect, it is essential to understand and develop NFT technology, which also appeals to end consumers with its structures and the opportunities it offers. Therefore, developing technical infrastructure and blockchain technology ensures NFT activity. Various model and system suggestions for these technologies, used in different sectors such as education, construction, and health, are shared above. On the other hand, security and compliance are also issues that NFTs offer and that users may still have concerns about. Among the prominent issues are ensuring tax control, introducing a socio-technical perspective on cyber security issues, using it as a security tool in digital production systems, and verifying personal information. Diffusion of innovations is possible by integrating them into corporate strategies and business models. At this point, issues such as developing business models specific to NFT technology and adapting them to the system as an active income source also come to the fore. Finally, the literature research showed that user experience also plays a role in NFT activity. Providing convenience to the user experience, addressing user concerns, and providing ease of system use may increase NFT activity. No studies have evaluated the prominent criteria for making NFT activity more effective by integrating other Web 3.0 technologies. In other words, it has been determined that a model is needed to evaluate the prominent criteria in studies to increase NFT effectiveness. To address this gap in the literature, a model proposal has been developed that helps identify critical factors for increasing NFT activity.

3. Methodology

The aim of the study is to determine the most appropriate strategy in choosing identity management for NFT in Metaverse. For this purpose, first, criteria for NFT management need to be determined and weighted. The criteria determined in the literature review are weighted with the M-SWARA method. In the second stage of the study, strategies are evaluated based on the criteria. This evaluation is carried out by the VIKOR method. The two-stage proposed method is carried out based on expert opinions. Therefore, it is recommended to consider fuzzy numbers in order to include uncertainty in linguistic expressions in the analysis. Therefore, QPFR numbers have been integrated into these two methods. The most current version for QPFR numbers is the form in which the golden ratio is used. In this way, it will be possible to handle uncertainty in the most realistic way. In addition, since the selection of experts and their equal importance is a matter criticized in the literature, artificial intelligence-based expert selection is recommended. The details of the proposed model are shown in Fig. 1.

3.1. AI-based decision-making for expert prioritization

In the dynamics of decision-making process, diversity in the demographic information and experience of experts has become important in recent years to be taken into account in the analysis process. However, difficulty in prioritizing experts and diversity also create complexity of analysis. The proposed solution for this complexity is the AI-based k-means clustering algorithm. Clusters are created based on basic demographic characteristics of experts such as education, salary, age and sector. In determining the optimal number of clusters, the elbow method is preferred. The elbow method is a strategic visualization technique in finding the effective point in determining the effectiveness of the model. After this, the weights of the clusters are defined. These weights reflect the sizes and diversity of the clusters. Finally, the weights of individual experts in each cluster are calculated proportionally based on their proximity to the cluster centre. With an artificial intelligence-based methodology, an innovative approach to expert prioritization is achieved. Artificial intelligence-based expert prioritization steps are presented below.

In the Step 1, the number of optimal clusters are determined using the elbow method. The elbow method, an important component of methodology of study, serves as a guidance for defining the optimal number of clusters in decision-maker grouping (Liu et al, 2024). According to plotting the Within-Cluster Sum of Squares (WCSS) against the number of clusters (k), this method systematically reveals the point at which additional clusters cease to significantly increase the effectiveness of the model, elbow-shaped in the graph. On the basis, the elbow technique enables us to reconcile the granularity of clustering with the declining rewards of further complexity by serving as a strategic navigator (Costantiello and Leogrande, 2024). This ensures that the selected number of clusters encapsulates the essential patterns within the demographic data, providing a robust foundation for subsequent steps in our prioritization methodology.

In Step 2, the Within-Cluster Sum of Squares (WCSS) is computed for the different values of k using Eq. (1).

$$WCSS = \sum_{j=1}^k \sum_{x_i \in C_j} d(x_i, c_j)^2$$

1

Where k is the number of clusters, C_j is the set of data points in cluster j , x_i is a data point, c_j is the cluster center of cluster j , and $d(x_i, c_j)$ is the Euclidean distance between x_i and c_j . The elbow is determined as an optimal k value where the reduction in WCSS slows down by plotting the values of WCSS and k .

In the Step 3 of the AI-based expert prioritization, the K-means clustering algorithm is used for clustering decision makers. The optimal k value is applied for initial cluster centres as $c_1, c_2, \dots, c_k$ and each data point x_i to the nearest cluster center is assigned to define the cluster assignments with Eq. (2).

$$d(x_i, x_j) = \sqrt{\sum_{l=1}^n (x_{il} - x_{jl})^2}$$

2

Where n represents the number of dimensions of the data. The cluster assignment of each data point x_i is denoted by a_i where $a_i = j$ is that x_i belongs to cluster j (Yang, 2024; Kong et al, 2024). Cluster centers are updated by taking the average of data points in each cluster by Eq. (3).

$$c_j = \frac{1}{|C_j|} \sum_{x_i \in C_j} x_i$$

3

where C_j means the set of data points in cluster j , and $|C_j|$ is the number of data points in cluster j . Eqs. (2) and (3) are applied iteratively until no data point changes its cluster assignment, or until a maximum number of iterations is reached.

In Step 4, the weights of the decision makers are calculated by considering the cluster weights of the decision makers. The mean standard deviations of each cluster are computed with the help of Equations (4)-(6).

$$s_j = \frac{1}{n} \sum_{l=1}^n \sigma_{jl}$$

4

$$\sigma_{jl} = \sqrt{\frac{1}{|C_j|} \sum_{x_i \in C_j} (x_{il} - x_{jl})^2}$$

5

$$x_{jl} = \frac{1}{|C_j|} \sum_{x_i \in C_j} x_{il}$$

6

where, s_j is the standard deviation of cluster j . n is the number of features or dimensions of the data, and σ_{jl} is the standard deviation of feature l in cluster j . x_{jl} is the mean of feature l in cluster j . Then, the cluster weights w_j are obtained with Eq. (7).

$$w_j = |C_j| \times s_j$$

7

where $|C_j|$ is the magnitude of cluster j (Lee, 2024). The weights of the decision makers are calculated with Eq. (8).

$$w_{tj} = \frac{1}{|C_j|} \frac{w_j}{\sum_{w_j \in C_j} w_j}$$

8

where, t means the number of decision makers w_{tj} represents the weight of decision maker t in cluster j .

3.2. Neuro Computing with Facial expressions

Facial expressions such as anger, happiness and sadness are important elements in human communication. For decision makers, this is also an effective factor in their decision making. The network of facial muscles forms our judgments about individuals based on their emotional expressions. Neuro Decision Making Concept is a new decision-making methodology for classifying facial expressions and emotions (Göller et al, 2024). The basis of the neurodecision-making method lies in Darwin's research on the non-verbal display of emotions. Facial action coding system (FACS) is a classification algorithm consisting of 46 different action units developed in the 1970s (Rahadian et al, 2024). The FACS algorithm is used as a reliable and valid method in areas requiring nonverbal behavior and social interaction (Westermann et al, 2024; Cakir and Arica, 2024).

3.3. Modelling Uncertainty with Quantum picture fuzzy rough sets with golden cuts

Quantum picture fuzzy rough sets with gold dashes are used to determine the lower and upper limits in the subjective evaluations of experts. Quantum mechanics and the golden ratio are used in the boundary determination process used here. Quantum mechanics is the branch of physics that investigates matter and energy at the subatomic level. The theory takes into account wave functions, quantum states (Du et al, 2024; Hou, 2024). Additionally, fuzzy number sets in mathematics are one of the set theories that take uncertainty into account. Fuzzy sets can create structures that contain uncertainty in analysis by defining the belonging of the elements with numbers between 0 and 1. In quantum mechanics, the uncertainty in the position of a massless particle is integrated into fuzzy number systems. The particle's ownership of momentum is proportional to the size of the wave function (Kou et al, 2023; Fan et al, 2023). The algorithm combining quantum mechanics, golden ratio and fuzzy numbers is given by Equations (9)-(11).

$$Q(|u\rangle) = \phi e^{i\theta}$$

9

$$|C\rangle = \{|u_1\rangle, |u_2\rangle, \dots, |u_n\rangle\}$$

10

$$\sum_{|u\rangle \subseteq |C\rangle} |Q(|u\rangle)| = 1$$

11

In Equations (9)–(11), C represents a collection of exhaustive events represented with $|u_i\rangle$. The squared magnitude of the wave function, $|Q(|u\rangle)| = \phi^2$, provides the amplitude-based result for the probability of occurrence of event $|u\rangle$ as determined. The value of ϕ^2 must lie within the range of 0 to 1, and θ^2 represents the phase angle of event $|u\rangle$. Furthermore, $|\phi_1|^2$ serves as the degree of belief in event $|u\rangle$, with θ representing its phase angle, which can range from 0 to 360 degrees (Carayannis et al, 2023).

Picture fuzzy sets (PFSs) is one of fuzzy sets and the intuitionistic fuzzy sets are determined in 1980s (Rani et al, 2024). This method is determined by Cuong and his colleague in the last decade to provide more comprehensive fuzzy-based evaluations for the complex decision-making problems (Ahmad et al, 2024). For this purpose, PFSs considers the positive, neutral, negative, refusal membership degrees of the fuzzy sets on a universe. The conventional fuzzy sets in Eq. (12) include the membership function.

$$A = \{ \langle x, \mu_A(x) \rangle | x \in X \}$$

12

Where A is a fuzzy set, X is a universe of discourse and μ_A represents the membership degree of x in the fuzzy set A . $\mu_A: X \rightarrow [0,1]$. However, intuitionistic fuzzy sets in Eq. (13) are formulated with membership and non-membership functions.

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in X \}$$

13

Where ν_A is the non-membership function of fuzzy set, $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. Picture fuzzy sets are shown in Eq. (14). The sets are by considering the additional function parameters of the universe X with the following statement.

$$A = \{ \langle x, \mu_A(x), n_A(x), \nu_A(x), h_A(x) \rangle | x \in X \}$$

14

Where n_A represents the neutral and h_A means the refusal degrees of membership function of x in A . $\mu_A(x) + n_A(x) + \nu_A(x) + h_A(x) = 1, \forall x \in X$. Picture fuzzy sets also answers the complex models including several types of expert opinions entitled the membership value μ_A for 'yes', neutral value n_A for 'abstain', non-membership value ν_A for 'no', the refusal value h_A for 'ignoring'. So that, it is possible to obtain results that are more consistent with the real world (Kahraman, 2024). Some operations of picture fuzzy set A and B are shown with Equations (15)–(19).

$A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $n_A(x) \leq n_B(x)$ and $\nu_A(x) \geq \nu_B(x), \forall x \in X$ (15)

$A = B$ if $A \subseteq B$ and $B \subseteq A$ (16)

$$A \cup B = \left\{ \left(x, \max(\mu_A(x), \mu_B(x)), \min(n_A(x), n_B(x)), \min(\nu_A(x), \nu_B(x)) \right) | x \in X \right\}$$

17

$$A \cap B = \left\{ \left(x, \min(\mu_A(x), \mu_B(x)), \min(n_A(x), n_B(x)), \max(\nu_A(x), \nu_B(x)) \right) | x \in X \right\}$$

18

$$coA = \bar{A} = \left\{ \left(x, \nu_A(x), n_A(x), \mu_A(x) \right) | x \in X \right\}$$

19

The purpose of the rough number scope is to reduce the subjective and ambiguous assessments of decision-making analysis. It comprises a rough boundary interval, upper and lower bounds. Lower $\left(\underline{Apr}(C_i) \right)$, upper $\left(\bar{Apr}(C_i) \right)$ approximation, and boundary region $\left(Bnd(C_i) \right)$ of C_i are presented in Equations (20)–(22).

$$\underline{Apr}(C_i) = \cup \left\{ Y \in X / R(Y) \leq C_i \right\}$$

20

$$\underline{Apr}(C_i) = \cup \{Y \in X / R(Y) \geq C_i\}$$

21

$$\underline{Bnd}(C_i) = \cup \{Y \in X / R(Y) \neq C_i\}$$

22

Where Y means an arbitrary object of the universe X , R is the set of N classes $(C_1, \dots, C_N)$. C represents the objects in X . $\forall Y \in X, C_i \in R$. However, lower

$\left(\underline{Lim}(C_i)\right)$, upper $\left(\overline{Lim}(C_i)\right)$ limits and the rough number $\left(RN(C_i)\right)$ of C_i are given by Equations (23)-(25).

$$\underline{Lim}(C_i) = \sqrt[N_L]{\prod_{i=1}^{N_L} Y \in \underline{Apr}(C_i)}$$

23

$$\overline{Lim}(C_i) = \sqrt[N_U]{\prod_{i=1}^{N_U} Y \in \overline{Apr}(C_i)}$$

24

$$RN(C_i) = [\underline{Lim}(C_i), \overline{Lim}(C_i)]$$

25

Where N_L and N_U are the number of objects for $\underline{Apr}(C_i)$ and $\overline{Apr}(C_i)$. In this study, golden-cut fuzzy rough numbers based on quantum mechanics are proposed to obtain more comprehensive and consistent results than classical fuzzy-based modelling. One of the important advantages of these numbers is that they can take into account both the evaluations of different experts and the opinions of experts with specific characteristics. Quantum picture fuzzy rough sets (QPFRS) represent the degree of membership of an element by a complex number. Formulations of membership value are expressed by Equations (26)-(44).

$$|C_A\rangle = \left\{ \begin{array}{l} \langle u, [\underline{Lim}(C_{i\mu_A}), \overline{Lim}(C_{i\mu_A})](u), [\underline{Lim}(C_{in_A}), \overline{Lim}(C_{in_A})](u), \\ [\underline{Lim}(C_{iv_A}), \overline{Lim}(C_{iv_A})](u), [\underline{Lim}(C_{ih_A}), \overline{Lim}(C_{ih_A})](u) \rangle | u \in 2^{|C_A|} \end{array} \right\}$$

26

Where, $C_{i\mu_A}$ defines the membership, C_{in_A} gives information about the neutral degree, C_{iv_A} is about the non-membership, and C_{ih_A} explains the refusal degrees and their definitions in the picture fuzzy rough numbers are given as follows

$$\underline{Lim}(C_{i\mu_A}) = \frac{1}{N_{L\mu_A}} \sum_{i=1}^{N_{L\mu_A}} Y \in \underline{Apr}(C_{i\mu_A})$$

27

$$\underline{Lim}(C_{in_A}) = \frac{1}{N_{Ln_A}} \sum_{i=1}^{N_{Ln_A}} Y \in \underline{Apr}(C_{in_A})$$

28

$$\underline{Lim}(C_{iv_A}) = \frac{1}{N_{Lv_A}} \sum_{i=1}^{N_{Lv_A}} Y \in \underline{Apr}(C_{iv_A})$$

29

$$\lim_{-} \left(C_{ih_A} \right) = \frac{1}{N_{Lh_A}} \sum_{i=1}^{N_{Lh_A}} Y \in \text{Apr} \left(C_{ih_A} \right)$$

30

$$\lim_{-} \left(C_{i\mu_A} \right) = \frac{1}{N_{U\mu_A}} \sum_{i=1}^{N_{U\mu_A}} Y \in \text{Apr} \left(C_{i\mu_A} \right)$$

31

$$\lim_{-} \left(C_{in_A} \right) = \frac{1}{N_{Un_A}} \sum_{i=1}^{N_{Un_A}} Y \in \text{Apr} \left(C_{in_A} \right)$$

32

$$\lim_{-} \left(C_{iv_A} \right) = \frac{1}{N_{Uv_A}} \sum_{i=1}^{N_{Uv_A}} Y \in \text{Apr} \left(C_{iv_A} \right)$$

33

$$\lim_{-} \left(C_{ih_A} \right) = \frac{1}{N_{U\pi_A}} \sum_{i=1}^{N_{U\pi_A}} Y \in \text{Apr} \left(C_{ih_A} \right)$$

34

Where $N_{L\mu_A}, N_{Lh_A}, N_{Lv_A}, N_{Lh_A}$ are the number of elements in $\text{Apr} \left(C_{i\mu_A} \right), \text{Apr} \left(C_{in_A} \right), \text{Apr} \left(C_{iv_A} \right), \text{Apr} \left(C_{ih_A} \right)$ respectively while $N_{U\mu_A}, N_{Un_A}, N_{Uv_A}, N_{Uh_A}$ are defined for $\text{Apr} \left(C_{i\mu_A} \right), \text{Apr} \left(C_{in_A} \right), \text{Apr} \left(C_{iv_A} \right), \text{Apr} \left(C_{ih_A} \right)$.

$$\text{Apr} \left(C_{i\mu_A} \right) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{i\mu_A} \right\}$$

35

$$\text{Apr} \left(C_{in_A} \right) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{in_A} \right\}$$

36

$$\text{Apr} \left(C_{iv_A} \right) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{iv_A} \right\}$$

37

$$\text{Apr} \left(C_{ih_A} \right) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{ih_A} \right\}$$

38

$$\text{Apr} \left(C_{i\mu_A} \right) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{i\mu_A} \right\}$$

39

$$\text{Apr} \left(C_{in_A} \right) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{in_A} \right\}$$

40

$$\bar{Apr}(C_{iv_A}) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{iv_A} \right\}$$

41

$$\bar{Apr}(C_{ih_A}) = \cup \left\{ Y \in X / \tilde{R}(Y) \leq C_{ih_A} \right\}$$

42

Where $\tilde{C}_i = (C_{i\mu_A}, C_{in_A}, C_{iv_A}, C_{ih_A})$ and $\tilde{R}$ is the collection of $\{\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_n\}$. $C_{i\mu_A}, C_{in_A}, C_{iv_A}, C_{ih_A}$ are the picture fuzzy sets of class $\tilde{C}_i$. However, the general formulation of the quantum picture fuzzy sets (QPFs) with the amplitude and the angle results of event is presented below.

$$C = [C_\mu \cdot \theta^{j2\pi \cdot \alpha}, C_n \cdot \theta^{j2\pi \cdot \gamma}, C_v \cdot \theta^{j2\pi \cdot \beta}, C_h \cdot \theta^{j2\pi \cdot T}]$$

43

$$\phi^2 = |C_\mu (|u_i\rangle)|$$

44

The amplitudes of quantum membership, neutral, non-membership, and refusal degrees, symbolized by C_μ, C_n, C_v and C_h respectively, are given with terms of phase angles α, γ, β, T . The amplitude of the membership value C_μ of QFS's is symbolized by ϕ^2 . The utilization of the golden ratio in multi-objective optimization problems allows for a harmonious balance between two opposing objectives, such as maximizing profit while minimizing risk. The golden ratio is approximately 1.618 and the value is symbolized by ϕ (phi). In the context of the membership and non-membership degrees, the golden ratio can be utilized to establish the ideal weighting between these two objectives. Therefore, a compromise is achieved between the components of fuzzy sets. The representation of the degrees by the golden ratio could be expressed through amplitude with Equations (45) and (46).

$$C_n = \frac{C_\mu}{G}$$

45

$$C_h = \frac{C_v}{G}$$

46

In conclusion, the phase angle for the suggested sets is obtained. The phase angle of the membership in realm of QPFS's is symbolized by the symbol α with the Equations (47)-(49).

$$\alpha = |C_\mu (|u_i\rangle)|$$

47

$$\gamma = \frac{\alpha}{G}$$

48

$$T = \frac{\beta}{G}$$

49

X_1 and X_2 mean two universes, and $\tilde{A}_C$ and $\tilde{B}_C$ respectively represented by

$$\begin{pmatrix} \left[\lim(C_{\mu_A}), \lim(C_{\mu_A}) \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{-A}}{2\pi} \right), \left(\frac{\alpha_A}{2\pi} \right) \right]}, \left[\lim(C_{n_A}), \lim(C_{n_A}) \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{-A}}{2\pi} \right), \left(\frac{\gamma_A}{2\pi} \right) \right]}, \\ \left[\lim(C_{v_A}), \lim(C_{v_A}) \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{-A}}{2\pi} \right), \left(\frac{\beta_A}{2\pi} \right) \right]}, \left[\lim(C_{h_A}), \lim(C_{h_A}) \right] e^{j2\pi \cdot \left[\left(\frac{T_{-A}}{2\pi} \right), \left(\frac{T_A}{2\pi} \right) \right]} \end{pmatrix},$$

$$\begin{pmatrix} \left[\lim(C_{\mu_B}), \lim(C_{\mu_B}) \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{-B}}{2\pi} \right), \left(\frac{\alpha_B}{2\pi} \right) \right]}, \left[\lim(C_{n_B}), \lim(C_{n_B}) \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{-B}}{2\pi} \right), \left(\frac{\gamma_B}{2\pi} \right) \right]}, \\ \left[\lim(C_{v_B}), \lim(C_{v_B}) \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{-B}}{2\pi} \right), \left(\frac{\beta_B}{2\pi} \right) \right]}, \left[\lim(C_{h_B}), \lim(C_{h_B}) \right] e^{j2\pi \cdot \left[\left(\frac{T_{-B}}{2\pi} \right), \left(\frac{T_B}{2\pi} \right) \right]} \end{pmatrix}$$

and, they are two QPFRS's derived from the universes of discourse X_1 and X_2 . The operations of QPFR numbers are detailed with Equations (50)-(53).

$$\lambda * \tilde{A}_C = \begin{pmatrix} \left[\lim(C_{\mu_A})\lambda, \lim(C_{\mu_A})\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{-A}}{2\pi} \right)\lambda, \left(\frac{\alpha_A}{2\pi} \right)\lambda \right]}, \left[\lim(C_{n_A})\lambda, \lim(C_{n_A})\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{-A}}{2\pi} \right)\lambda, \left(\frac{\gamma_A}{2\pi} \right)\lambda \right]}, \\ \left[\lim(C_{v_A})\lambda, \lim(C_{v_A})\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{-A}}{2\pi} \right)\lambda, \left(\frac{\beta_A}{2\pi} \right)\lambda \right]}, \left[\lim(C_{h_A})\lambda, \lim(C_{h_A})\lambda \right] e^{j2\pi \cdot \left[\left(\frac{T_{-A}}{2\pi} \right)\lambda, \left(\frac{T_A}{2\pi} \right)\lambda \right]} \end{pmatrix}$$

$$\lambda > 0$$

50

$$\tilde{A}_C^\lambda = \begin{pmatrix} \left[\lim(C_{\mu_A})^\lambda, \lim(C_{\mu_A})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{-A}}{2\pi} \right)^\lambda, \left(\frac{\alpha_A}{2\pi} \right)^\lambda \right]}, \left[\lim(C_{n_A})^\lambda, \lim(C_{n_A})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{-A}}{2\pi} \right)^\lambda, \left(\frac{\gamma_A}{2\pi} \right)^\lambda \right]}, \\ \left[\lim(C_{v_A})^\lambda, \lim(C_{v_A})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{-A}}{2\pi} \right)^\lambda, \left(\frac{\beta_A}{2\pi} \right)^\lambda \right]}, \left[\lim(C_{h_A})^\lambda, \lim(C_{h_A})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{T_{-A}}{2\pi} \right)^\lambda, \left(\frac{T_A}{2\pi} \right)^\lambda \right]} \end{pmatrix}$$

$$\lambda > 0$$

51

$$\tilde{A}_c \cup \tilde{B}_c = \left[\begin{array}{l} \left[\min \left(\lim_{\alpha} \left(C_{\mu_A} \right) e^{j2\pi \cdot \left(\frac{\alpha_-}{2\pi} \right)}, \lim_{\alpha} \left(C_{\mu_B} \right) e^{j2\pi \cdot \left(\frac{\alpha_-}{2\pi} \right)} \right), \max \left(\lim_{\alpha} \left(C_{\mu_A} \right) e^{j2\pi \cdot \left(\frac{\alpha_+}{2\pi} \right)}, \lim_{\alpha} \left(C_{\mu_B} \right) e^{j2\pi \cdot \left(\frac{\alpha_+}{2\pi} \right)} \right) \right], \\ \left[\min \left(\lim_{\gamma} \left(C_{n_A} \right) e^{j2\pi \cdot \left(\frac{\gamma_-}{2\pi} \right)}, \lim_{\gamma} \left(C_{n_B} \right) e^{j2\pi \cdot \left(\frac{\gamma_-}{2\pi} \right)} \right), \max \left(\lim_{\gamma} \left(C_{n_A} \right) e^{j2\pi \cdot \left(\frac{\gamma_+}{2\pi} \right)}, \lim_{\gamma} \left(C_{n_B} \right) e^{j2\pi \cdot \left(\frac{\gamma_+}{2\pi} \right)} \right) \right], \\ \left[\min \left(\lim_{\beta} \left(C_{v_A} \right) e^{j2\pi \cdot \left(\frac{\beta_-}{2\pi} \right)}, \lim_{\beta} \left(C_{v_B} \right) e^{j2\pi \cdot \left(\frac{\beta_-}{2\pi} \right)} \right), \max \left(\lim_{\beta} \left(C_{v_A} \right) e^{j2\pi \cdot \left(\frac{\beta_+}{2\pi} \right)}, \lim_{\beta} \left(C_{v_B} \right) e^{j2\pi \cdot \left(\frac{\beta_+}{2\pi} \right)} \right) \right], \\ \left[\min \left(\lim_{\tau} \left(C_{h_A} \right) e^{j2\pi \cdot \left(\frac{\tau_-}{2\pi} \right)}, \lim_{\tau} \left(C_{h_B} \right) e^{j2\pi \cdot \left(\frac{\tau_-}{2\pi} \right)} \right), \max \left(\lim_{\tau} \left(C_{h_A} \right) e^{j2\pi \cdot \left(\frac{\tau_+}{2\pi} \right)}, \lim_{\tau} \left(C_{h_B} \right) e^{j2\pi \cdot \left(\frac{\tau_+}{2\pi} \right)} \right) \right] \end{array} \right]$$

52

$$\tilde{A}_c \cap \tilde{B}_c = \left[\begin{array}{l} \left[\max \left(\lim_{\alpha} \left(C_{\mu_A} \right) e^{j2\pi \cdot \left(\frac{\alpha_-}{2\pi} \right)}, \lim_{\alpha} \left(C_{\mu_B} \right) e^{j2\pi \cdot \left(\frac{\alpha_-}{2\pi} \right)} \right), \min \left(\lim_{\alpha} \left(C_{\mu_A} \right) e^{j2\pi \cdot \left(\frac{\alpha_+}{2\pi} \right)}, \lim_{\alpha} \left(C_{\mu_B} \right) e^{j2\pi \cdot \left(\frac{\alpha_+}{2\pi} \right)} \right) \right], \\ \left[\max \left(\lim_{\gamma} \left(C_{n_A} \right) e^{j2\pi \cdot \left(\frac{\gamma_-}{2\pi} \right)}, \lim_{\gamma} \left(C_{n_B} \right) e^{j2\pi \cdot \left(\frac{\gamma_-}{2\pi} \right)} \right), \min \left(\lim_{\gamma} \left(C_{n_A} \right) e^{j2\pi \cdot \left(\frac{\gamma_+}{2\pi} \right)}, \lim_{\gamma} \left(C_{n_B} \right) e^{j2\pi \cdot \left(\frac{\gamma_+}{2\pi} \right)} \right) \right], \\ \left[\max \left(\lim_{\beta} \left(C_{v_A} \right) e^{j2\pi \cdot \left(\frac{\beta_-}{2\pi} \right)}, \lim_{\beta} \left(C_{v_B} \right) e^{j2\pi \cdot \left(\frac{\beta_-}{2\pi} \right)} \right), \min \left(\lim_{\beta} \left(C_{v_A} \right) e^{j2\pi \cdot \left(\frac{\beta_+}{2\pi} \right)}, \lim_{\beta} \left(C_{v_B} \right) e^{j2\pi \cdot \left(\frac{\beta_+}{2\pi} \right)} \right) \right], \\ \left[\max \left(\lim_{\tau} \left(C_{h_A} \right) e^{j2\pi \cdot \left(\frac{\tau_-}{2\pi} \right)}, \lim_{\tau} \left(C_{h_B} \right) e^{j2\pi \cdot \left(\frac{\tau_-}{2\pi} \right)} \right), \min \left(\lim_{\tau} \left(C_{h_A} \right) e^{j2\pi \cdot \left(\frac{\tau_+}{2\pi} \right)}, \lim_{\tau} \left(C_{h_B} \right) e^{j2\pi \cdot \left(\frac{\tau_+}{2\pi} \right)} \right) \right] \end{array} \right]$$

53

3.4. M-SWARA with Quantum picture fuzzy rough sets

SWARA method is referred to as the progressive weighting method in the multi-criterion decision-making literature. The basis of the SWARA method is the establishment of a hierarchical proportional structure in the evaluation of criteria. M-SWARA is an extension of the SWARA method, and its details are presented under this title (Rahadian et al, 2024; Mikhaylov et al, 2024).

Step 5 covers defining the criteria set for NFT management. Criteria affecting NFT management are defined from literature. In Step 6, dependency degrees between the criteria are defined from experts. Linguistic opinion is obtained to create relation matrix. Step 7 involves constructing QPFMs for the relationship matrix. Quantum picture fuzzy rough relation matrix is formulated by considering the linguistic evaluations of decision makers and the quantum spherical fuzzy numbers. $C_{\mu_A} \tilde{C} = [C_{ij}]_{n \times n}$ are the relationship each criterion. C_{ij} is about influence value of i-criterion over the j-criterion. The detail of matrix is given by Eq. (54).

$$C_k = \begin{bmatrix} 0 & C_{12} & \cdots & \cdots & C_{1n} \\ C_{21} & 0 & \cdots & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & \cdots & 0 \end{bmatrix}$$

54

Where C determines QPFR direct relation matrix.

$$C_{ij} = \left(\begin{array}{l} [Lim(C_{\mu_{ij}}), Lim(C_{\mu_{ij}})]e^{i2\pi \cdot [\frac{\alpha_{-ij}}{2\pi}], [\frac{\alpha_{ij}}{2\pi}]} , [Lim(C_{n_{ij}}), Lim(C_{n_{ij}})]e^{i2\pi \cdot [\frac{\gamma_{-ij}}{2\pi}], [\frac{\gamma_{ij}}{2\pi}]} , \\ [Lim(C_{v_{ij}}), Lim(C_{v_{ij}})]e^{i2\pi \cdot [\frac{\beta_{-ij}}{2\pi}], [\frac{\beta_{ij}}{2\pi}]} , [Lim(C_{h_{ij}}), Lim(C_{h_{ij}})]e^{i2\pi \cdot [\frac{T_{-ij}}{2\pi}], [\frac{T_{ij}}{2\pi}]} \end{array} \right), \text{ and } k \text{ is the number of decision}$$

makers. Step 8 is about defining QPFRs's for the relationship matrix. The C matrix of the experts is calculated with QPFR numbers in Eq. (55).

$$C = \left(\begin{array}{l} [min_{i=1}^k (Lim(C_{\mu_{ij}})), max_{i=1}^k (Lim(C_{\mu_{ij}}))]e^{i2\pi \cdot [min_{i=1}^k (\frac{\alpha_{-ij}}{2\pi}), max_{i=1}^k (\frac{\alpha_{ij}}{2\pi})]} , \\ [min_{i=1}^k (Lim(C_{n_{ij}})), max_{i=1}^k (Lim(C_{n_{ij}}))]e^{i2\pi \cdot [min_{i=1}^k (\frac{\gamma_{-ij}}{2\pi}), max_{i=1}^k (\frac{\gamma_{ij}}{2\pi})]} , \\ [min_{i=1}^k (Lim(C_{v_{ij}})), max_{i=1}^k (Lim(C_{v_{ij}}))]e^{i2\pi \cdot [min_{i=1}^k (\frac{\beta_{-ij}}{2\pi}), max_{i=1}^k (\frac{\beta_{ij}}{2\pi})]} , \\ [min_{i=1}^k (Lim(C_{h_{ij}})), max_{i=1}^k (Lim(C_{h_{ij}}))]e^{i2\pi \cdot [min_{i=1}^k (\frac{T_{-ij}}{2\pi}), max_{i=1}^k (\frac{T_{ij}}{2\pi})]} \end{array} \right)$$

55

In Step 9, defuzzified values are calculated. The $Defc$ of QPFRs's are calculated with Eq. (56).

$$\left(\begin{array}{l} Lim(C_{\mu_i}) - Lim(C_{n_i}) + Lim(C_{\mu_i}) \cdot (Lim(C_{v_i}) - Lim(C_{h_i})) + \left(\frac{\alpha_{-ij}}{2\pi} \right) - \left(\frac{\gamma_{-ij}}{2\pi} \right) + \left(\frac{\alpha_{-ij}}{2\pi} \right) \cdot \left(\left(\frac{\beta_{-ij}}{2\pi} \right) - \left(\frac{T_{-ij}}{2\pi} \right) \right) + \\ Lim(C_{\mu_i}) - Lim(C_{n_i}) + Lim(C_{\mu_i}) \cdot (Lim(C_{v_i}) - Lim(C_{h_i})) + \left(\frac{\alpha_{ij}}{2\pi} \right) - \left(\frac{\gamma_{ij}}{2\pi} \right) + \left(\frac{\alpha_{ij}}{2\pi} \right) \cdot \left(\left(\frac{\beta_{ij}}{2\pi} \right) - \left(\frac{T_{ij}}{2\pi} \right) \right) \end{array} \right)$$

$$Defc_i = \frac{\quad}{2}$$

56

In Step 10, the normalized relationship matrix is obtained. Then, in Step 11, it continues with the calculation of s_j , k_j , q_j and w_j values for the relationship value of each criterion.

$$k_j = \begin{cases} 1 & j = 1 \\ s_j + 1 & j > 1 \end{cases}$$

57

$$q_j = \begin{cases} 1 & j = 1 \\ \frac{q_{j-1}}{k_j} & j > 1 \end{cases}$$

58

$$\text{If } s_{j-1} = s_j, q_{j-1} = q_j; \text{ If } s_j = 0, k_{j-1} = k_j$$

$$w_j = \frac{q_j}{\sum_{k=1}^n q_k}$$

59

s_j means the comparative importance rate by QPFRS's and provides the importance value of the criterion c_j on the following criterion c_{j+1} . k_j represents the coefficient value of the s_j and q_j defines the recalculated weight of k_j . w_j gives knowledge about the weights of the criteria in the fuzzy sets. The values are sorted. Step 12 involves constructing the relation matrix and the directions between the criteria. Stable values of relation matrix with the values of w_j is obtained by transposing and limiting the matrix to the power of $2t+1$. t is an arbitrarily biggest. Thus, the weighting results of w_j are obtained with the stabilization process in the M-SWARA method. Impact-relation degrees of the criteria are constructed by the threshold value that is the average value of relation matrix. Greater values than the threshold in the column is influenced by the criteria in the row (Mikhaylov et al, 2023). So, the impact directions of the criteria can be showed properly.

3.5. VIKOR with Quantum picture fuzzy rough sets

VIKOR is a decision-making method based on closeness to the ideal solution through consensus. The VIKOR method aims to rank the alternatives, considering the criteria. Compromise solution in the VIKOR method implies consensus with mutual concessions (Hu et al, 2024; Razzaque et al, 2024). The step of the extended VIKOR method is given below.

In Step 13, emotional expressions are collected for the alternatives. Linguistic evaluation is obtained. In Step 14 is about constructing the QPFNs for the decision matrix. In Step 15, QPFRS sets are determined for the decision matrix. The decision matrix is defined as X , with X_{ij} representing the alternative i with respect to criterion j based on the QPFRNs-based linguistic opinions of experts. The matrix is illustrated by Eq. (60).

$$X_k = \begin{bmatrix} 0 & X_{12} & \cdots & \cdots & X_{1m} \\ X_{21} & 0 & \cdots & \cdots & X_{2m} \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & \cdots & 0 \end{bmatrix}$$

60

Step 16 involves computing the defuzzified decision values. The defuzzified of the QSFS's for the decision matrix are obtained with the help of Eq. (56). In

Step 17, S_i , R_i and Q_i values are constructed. First, f_j values need to be calculated. The best $\tilde{f}_j^*$ and worst $\tilde{f}_j^-$ values for each criterion function are calculated by formula (61).

$$\tilde{f}_j^* = \max_i \tilde{x}_{ij} \text{ and } \tilde{f}_j^- = \min_i \tilde{x}_{ij}$$

61

The mean group utility and maximal regret are calculated with Equations (62)–(63).

$$\tilde{S}_i = \sum_{j=1}^n \tilde{w}_j \frac{\left(\left| \tilde{f}_j^* - \tilde{x}_{ij} \right| \right)}{\left(\left| \tilde{f}_j^* - \tilde{f}_j^- \right| \right)}$$

62

$$\tilde{R}_i = \max_j \left[\tilde{w}_j \frac{\left(\left| \tilde{f}_j^* - \tilde{x}_{ij} \right| \right)}{\left(\left| \tilde{f}_j^* - \tilde{f}_j^- \right| \right)} \right]$$

63

The final scores of alternatives is obtained using Eq. (64).

$$\tilde{Q}_i = v \left(\tilde{S}_i - \tilde{S}^* \right) / \left(\tilde{S}^- - \tilde{S}^* \right) + (1 - v) \left(\tilde{R}_i - \tilde{R}^* \right) / \left(\tilde{R}^- - \tilde{R}^* \right)$$

64

The weight of the strategy of maximum group utility is $v = 0.5$. The weight of individual regret means $(1 - v)$. Each case is considered. The final ranking of alternatives requires the fulfillment of two requirements once the values of S, R, and Q are sorted. Condition 1 is calculated with Eq. (65). The situation considers the second position of the alternatives ranked according to Q value. Next condition requires that the alternative must be ranked by either S or R, or both.

$$Q(A^{(2)}) - Q(A^{(1)}) \geq 1 / (j - 1)$$

65

If any of these conditions are not met, a compromise solution is preferred. These alternatives are preferred based on their close ranking scores. It is calculated with the maximum value of M (Yue, 20024). In Step 18, comparative raking values are calculated for consistency of results with sensitivity analysis.

4. Analysis

In this section, the findings obtained in determining strategies for NFT management in the Metaverse are presented under subtitles.

4.1. Prioritizing the expert choices with AI-based decision-making method

Five experts with academic or industry experience are determined. Artificial intelligence is used to determine the people to be included in the analysis among these experts. Firstly, in Step 1, the demographic information of the experts is defined. The demographic information of the experts is displayed in Table 1.

Table 1
Specifications of the decision makers

Decision Maker	Education	Experience (year)	Salary (USD)	Age	Industry
DM1	1 (Bachelor)	24	3000	46	1 (Service)
DM2	2 (Master)	20	4000	53	2 (Production)
DM3	3 (PhD)	17	4500	42	3 (Education)
DM4	3 (PhD)	15	4000	44	3 (Education)
DM5	2 (Master)	15	2500	46	2 (Production)

In Step 2, the optimal k value is calculated for cluster analysis using Eq. (1). The values of the WCSS are computed for the different number of clusters. In this case, the number of cluster is 5, from 1 to 5, the set of WCSS values are presented for the different k values as seen in Table A1. Also, the elbow point is defined for selecting the optimal number of clusters for the dataset. Accordingly, Fig. 2 illustrate the plot of the values of WCSS against the number of k.

According to the results, for K = 3, the WCSS value is minimized and optimized as adding more clusters will not significantly reduce the WCSS value. In Step 3, k-means clustering algorithm is applied for decision makers with Equations (2)-(3). The optimal value for K number is applied for defining the clusters of decision makers. In this example, the optimal value of K is 3. The iteration results of different 3 clusters are given in Table A2. As seen in Table A2, the cluster assignment results are same with initial cluster centers and average of data points for iteration 1. So, the cluster of the decision makers is considered as DM1 and DM5 are considered in cluster 1; DM2 and DM4 are listed in cluster 2; DM3 is stated in cluster 3. In Step 4, the weights of the decision makers are computed by considering the cluster weights of the decision makers in Table A3. The weights of the decision are calculated with the help of Equations (4)-(8). The weights of the decision makers for expert prioritization are given in Table 2.

Table 2
Weights of the decision makers

Decision Maker	Weights
DM1	.48
DM2	.02
DM3	.00
DM4	.02
DM5	.48

In Table 2, it is seen that DM1 and DM5 have the best priorities with the value of 0.48 as DM2 and DM4 have relatively the weakest priorities among the decision makers. However, DM3 has no priority in the expert team. Thus, in the decision-making process, the decision makers excluding DM3 are considered for collecting their linguistic evaluations for assessing the criteria and alternatives.

4.2. Weighting the criteria set for managing Non-Fungible Tokens with QPFR-M-SWARA

Step 5 is about defining the criteria set for NFT management. As a result of the literature review, four criteria that are effective on managing NFT are determined. The determined criteria are given in Table 3.

Table 3
Criteria set for managing Non-Fungible Tokens

Criteria	Code
C1: User Experience	EXP
C2: Security and Compliance	SEC
C3: Monetization and Business Strategy	MBS
C4: Technical and Blockchain Infrastructure	TBI

Step 6 involves collecting emotional expressions for the criteria. In this process, opinions are taken from determined experts. The emotional expressions are shown in Table A4. Action unit combinations is shown in Table 4.

Table 4
Observed action unit combinations of
emotional expressions

DM1				
	EXP	SEC	MBS	TBI
EXP		(5,25)	(7,25)	(1,12)
SEC	(5,25)		(7,25)	(5,25)
MBS	(7,5)	(7,25)		(1,12)
TBI	(7,5)	(5,25)	(14,6)	
DM2				
	EXP	SEC	MBS	TBI
EXP		(1,12)	(5,25)	(5,25)
SEC	(5,25)		(7,25)	(5,25)
MBS	(7,5)	(7,25)		(1,12)
TBI	(14,6)	(1,12)	(14,6)	
DM4				
	EXP	SEC	MBS	TBI
EXP		(5,25)	(7,25)	(7,25)
SEC	(1,12)		(7,5)	(5,25)
MBS	(7,5)	(7,5)		(14,6)
TBI	(7,5)	(5,25)	(7,25)	
DM5				
	EXP	SEC	MBS	TBI
EXP		(5,25)	(7,25)	(5,25)
SEC	(6,26)		(7,25)	(6,26)
MBS	(14,6)	(14,6)		(1,12)
TBI	(7,25)	(1,12)	(7,25)	

In Step 7, QPFN for the relation matrix is constructed using Eq. (54). The detail of result is given in Table A5. In Step 8, the quantum picture fuzzy rough sets for the relation matrix are determined with Eq. (55). Table A6 presents the relevant results. Step 9 is the step defuzzified the values for the criteria using Eq. (56). The defuzzified values is shown in Table A7. In Step 10, the relationship matrix is normalized with Equations (57)-(59). The results are exhibited in Table A8. In Step 11, s_j , k_j , q_j , and w_j values are calculated. The values are given in Table A9. In Step 12, relation matrix and the directions among the criteria are constructed. The details of results are presented in Table A10. Finally, the matrix is stabilized. The result is shown in Table 5.

Table 5
Stable Matrix

	EXP	SEC	MBS	TBI
EXP	.233	.233	.233	.233
SEC	.275	.274	.275	.275
MBS	.231	.231	.231	.231
TBI	.261	.261	.261	.261

According to Table 5, the most important criterion is SEC, while the second most important criterion is TBI. Because the Stable matrix values of these two criteria are the highest. In the order of importance of the criteria, MBS ranks last.

4.3. Ranking the identity management choices for Non-Fungible Tokens in the Metaverse with QPFR-VIKOR

In Step 13, emotional expressions for the alternatives are collected. For this purpose, alternatives are determined first. The alternatives considered in the study are presented with their codes in Table 6.

Table 6
Strategies of managing NFT in Metaverse

Alternatives	Codes
A1: Biometrics for unique identification	BIO
A2: User verification using real identities	VER
A3: Decentralization by removing central authorities	DEC
A4: Privacy with authentication	PRI

Then, the observed unit combinations of emotional expressions for the alternatives are obtained using Eq. (60). The obtained values are presented in Table 7.

Table 7
Observed action unit combinations of emotional
expressions for the alternatives

DM1				
	EXP	SEC	MBS	TBI
BIO	(6,12)	(27,26)	(7,6)	(1,2)
VER	(27,26)	(2,25)	(7,6)	(15,1)
DEC	(1,2)	(15,1)	(15,1)	(14,27)
PRI	(2,25)	(2,25)	(1,2)	(27,26)
DM2				
	EXP	SEC	MBS	TBI
BIO	(2,25)	(27,26)	(14,25)	(1,2)
VER	(2,25)	(27,26)	(7,6)	(1,2)
DEC	(7,6)	(14,27)	(14,27)	(15,1)
PRI	(1,25)	(27,26)	(14,25)	(7,6)
DM4				
	EXP	SEC	MBS	TBI
BIO	(1,25)	(27,26)	(27,26)	(1,2)
VER	(2,25)	(2,25)	(14,25)	(14,27)
DEC	(7,6)	(7,6)	(14,27)	(15,27)
PRI	(1,2)	(1,2)	(1,2)	(1,2)
DM5				
	EXP	SEC	MBS	TBI
BIO	(6,12)	(27,26)	(1,2)	(1,2)
VER	(2,25)	(27,26)	(14,25)	(1,2)
DEC	(2,25)	(7,6)	(1,2)	(7,6)
PRI	(1,25)	(2,25)	(1,2)	(27,26)

In Step 14, QPFN for the decision matrix is constructed in Table A11. Step 15 involves determining the quantum picture fuzzy rough sets for the decision matrix. The set is given in Table A12. With Step 16, the defuzzified decision values in Table A13 are calculated. In Step 17, S, R and Q values are constructed using Equations (61)-(65). The value is given in Table A14. Step 18 is about comparison of results with sensitivity analysis. In this step, the results of two methods are compared with four different cases. The results obtained are shared in Table 8.

Table 8
Comparative ranking values with sensitivity analysis

Extended VIKOR (v:0.5)				
	Case 1	Case 2	Case 3	Case 4
BIO	1	1	1	1
VER	3	3	3	3
DEC	4	4	4	4
PRI	2	2	2	2
Extended TOPSIS				
	Case 1	Case 2	Case 3	Case 4
BIO	1	1	1	1
VER	3	3	3	3
DEC	4	4	4	4
PRI	2	2	2	2

In case of maximum group utility, veto, and consensus, biometrics for unique identification has the best ranking performance among the alternatives. Privacy with authentication plays also critical role for the effectiveness of this process.

5. Discussion

It has been determined that security must be ensured first to increase the use of non-fungible tokens on the Metaverse platform. A secure platform allows users to feel safe regarding these tokens. In other words, if personal data can be securely protected, investors' anxiety will decrease, and they will prefer this platform more. Moreover, non-fungible tokens can represent assets of very high value. Thanks to the security of the platform, the risk of theft of these assets can be minimized. In addition, these tokens also contribute to ensuring payment security. These issues allow investors to prefer this platform more. In this context, it is appropriate to take some precautions to increase security. For example, authentication methods need to be strengthened to ensure the security of user accounts. Cruz et al. (2024) mentioned that the application of a double control system helps minimize the risks in the process. On the other hand, Lekhi (2024) underlined that security audits should be performed on the system at regular intervals. As a result of the findings obtained in these inspections, more effective security measures should be designed. Similarly, Kizza (2024) identified that educating users on security issues raises awareness about these measures. This situation helps to significantly reduce security problems.

It is also identified that the technological infrastructure must be sufficient to increase the use of non-fungible tokens on the Metaverse platform. High technological infrastructure allows faster solutions to possible problems. On the other hand, businesses must have sufficient technological infrastructure so that many people can operate smoothly on this platform. Moreover, the development of technological infrastructure helps to implement transactions on the blockchain system more securely. This allows the trading volume on the platform to be increased. Due to these issues, businesses need to take some actions to improve the technological infrastructure. In this context, Messina et al. (2024) concluded that performance analysis should be carried out by performing regular checks on the platform. As a result of these analyses, necessary steps can be taken to use more up-to-date technology. Additionally, Alt et al. (2024) and Pineda et al. (2024) stated that appropriate mobile applications should be developed so that users can access the platform via mobile devices. This situation allows more investors to trade on this platform.

6. Conclusion

In this study, it is aimed to identify the appropriate the identity management choices of non-fungible tokens in the Metaverse. There are three different stages in the proposed novel fuzzy decision-making model. The first stage includes prioritizing the expert choices with artificial intelligence-based decision-making methodology. In the second stage, the criteria sets for managing non-fungible tokens are weighted by using Quantum picture fuzzy rough sets-based M-SWARA methodology. Thirdly, the identity management choices regarding non-fungible tokens in the Metaverse are ranked with Quantum picture fuzzy rough sets oriented VIKOR. It is concluded that security must be ensured first to increase the use of non-fungible tokens on the Metaverse platform. Furthermore, technological infrastructure must also be sufficient to achieve this objective. Moreover, biometrics for unique identification has the best ranking performance among the alternatives. Privacy with authentication plays also critical role for the effectiveness of this process.

The main contribution of this study is that artificial intelligence methodology is integrated to the fuzzy decision-making modelling to differentiate the experts. With the help of this situation, it can be possible to create clusters for the experts. Hence, the opinions of experts outside this group may be excluded from the scope. Similarly, M-SWARA methodology is preferred to weight the determinants. Owing to this issue, the causal directions among these

items can be considered. The main limitation of this study is making a general evaluation with respect to the effectiveness of the non-fungible tokens in Metaverse platform. Thus, in the following studies, an industry-specific examination can also be performed. With the help of these analysis results, more specific strategies can be presented for these industries. On the other side, there are also some limitations in the proposed model. In this study, artificial intelligence methodology is integrated to the fuzzy decision-making modelling to differentiate experts. However, the weights of these experts are not calculated. Therefore, for the future research direction, these weights can also be identified by considering artificial intelligence theory.

Declarations

Conflict of Interest

About the publication of this manuscript the authors declare that they have no conflict of interest.

Ethics Declaration Statement

The authors state that this is their original work and it is neither submitted nor under consideration in any other journal simultaneously.

Human and Animal Participants:

This article does not contain any studies with human participants or animals performed by any of the authors.

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Author Contribution

Gang Kou: Conceptualization, Investigation, Methodology, Software, Validation, Formal analysis, Visualization, Data Curation, Writing - Original Draft, Review & Editing. Hasan Dinçer: Investigation, Project administration, Writing - Original Draft, Review & Editing. Dragan Pamucar: Visualization, Data Curation, Writing - Original Draft, Review & Editing. Serhat Yüksel: Resources, Visualization, Writing - Original Draft, Review & Editing. Muhammet Deveci: Investigation, Project administration, Writing - Original Draft, Review & Editing. Gabriela Oana Olaru: Resources, Visualization, Writing - Original Draft, Review & Editing. Serkan Eti: Investigation, Project administration, Writing - Original Draft, Review & Editing.

Data Availability

The data will be available on reasonable request.

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Figures

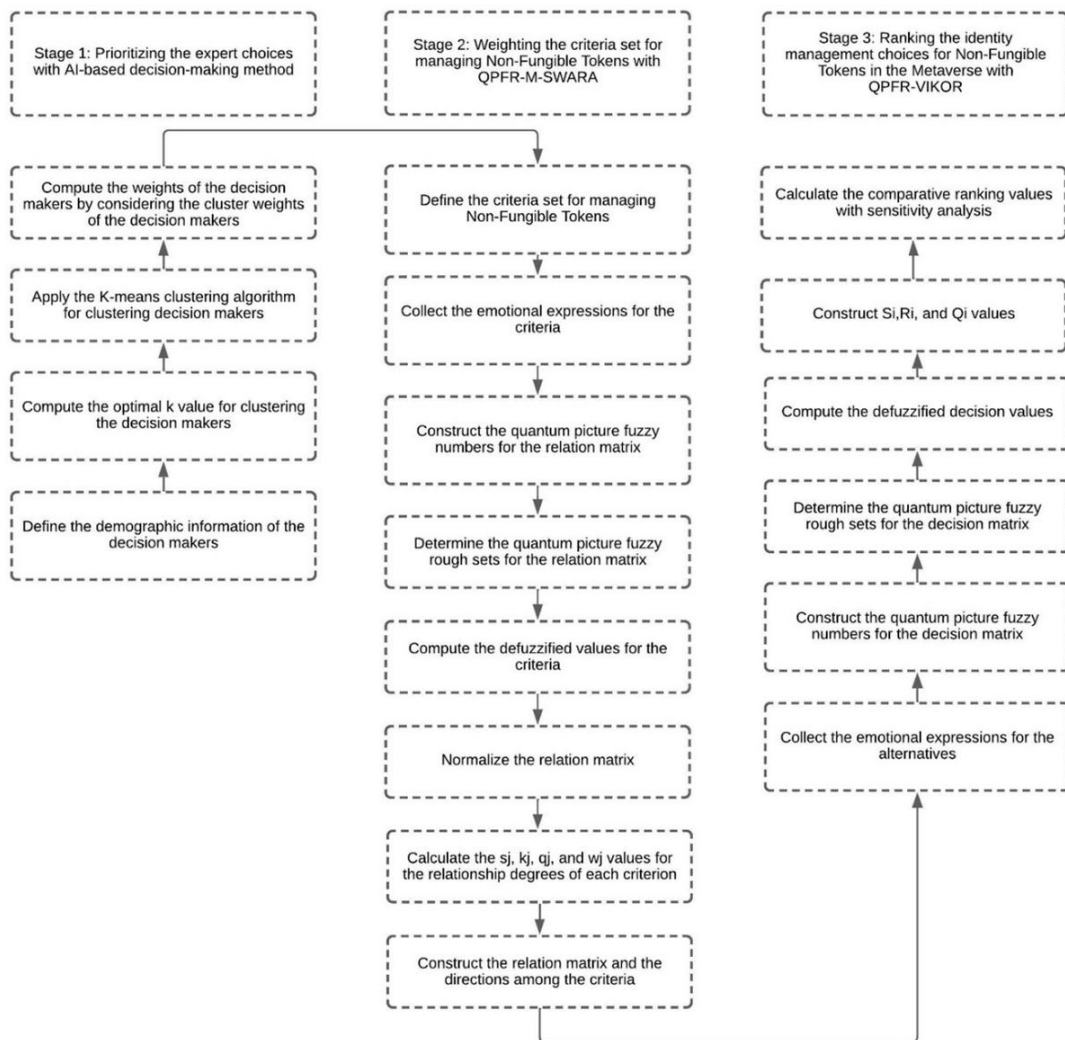


Figure 1

The flowchart of hybrid model

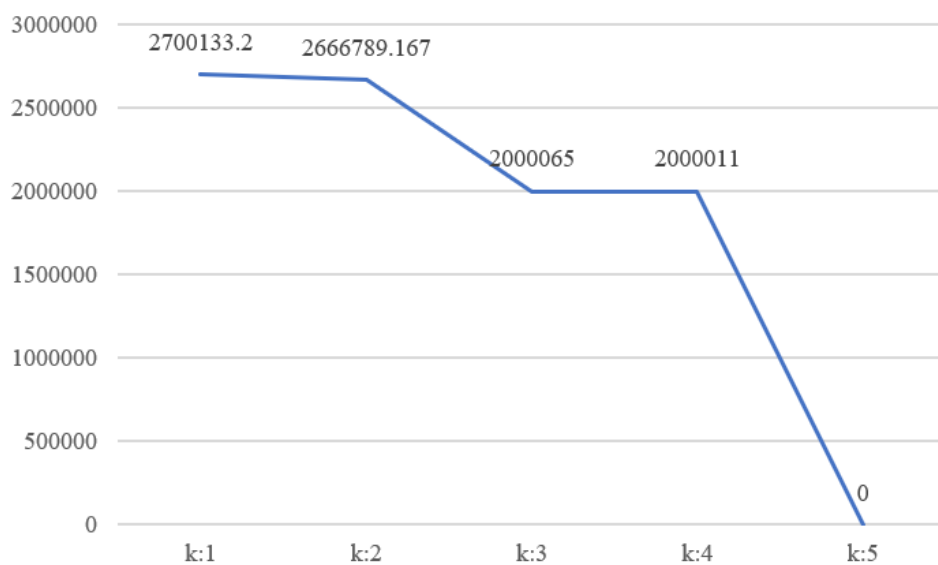


Figure 2

The plot of the WCSS values and k numbers

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